



PRESSURE WAVE PROPAGATION IN A POROUS MEDIUM IN THE CASE OF A NONLINEAR FILTRATION LAW

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The propagation of a pressure wave in a reservoir is considered taking into account the compressibility of both the rock itself and the fluid. In this case, two cases are considered: a linear filtration law - Darcy's law and a generalized filtration law in the form of Forchheimer. Graphs of the distribution of pressure in time and space in the process of shock-wave action with a sharp increase in pressure are obtained.

Keywords: shock wave, rock deformation, nonlinear Forchheimer law

INTRODUCTION

The study of the impact of shock waves and vibrations on the soil, and the fluid-filled reservoirs located in it, is of great importance for the efficient extraction of useful resources. For example, it is proposed to use low-power explosions from the surface to change the porous and/or fractured structure of the reservoir to release a large amount of oil, which is inaccessible [1]. If a stable compressive pressure consolidates the rock and its permeability decreases, then in the case of pressure waves passing through the rock, the picture becomes somewhat more complicated. Each point in space experiences sign-variable pressure, and depending on each specific case and the structure of the reservoir, additional cracking, partial destruction can occur - which increases the pore clearance and, accordingly, the permeability of the rock, and in another case, vice versa, somewhat locally compact the rock, since it can remove some small particles of the medium from unstable equilibrium. The propagation of a shock wave in solid media, including ground rocks, is often accompanied by high compressive forces on the entire structure of the material, along with any inhomogeneities and inclusions that may be contained in it. During the propagation of shock disturbances in the soil mass, many characteristic areas arise that differ from each other in the deformation mechanisms implemented in this process.[2]. A.Yu. Illinsky proposed a simplified soil model[3], on the basis of which A.P. Sinitsin obtained an equation for determining the law of propagation of the front of a shock wave propagating $x = S(t)$ in undisturbed soil without taking into account elastic perturbations [4]. In turn, U.S. Sarimsakov, A.I. Babichev, and I.T. Elmuratov generalized this law at a pressure value greater than a certain critical value for a given soil when $P > P^*$, and the soil with initial density ρ_0 and elastic wave propagation velocity α_0 is pressed to a new state. They came up with a formula like:

$$P(t) = \rho_0 \theta_* \frac{d(SS)}{dt} + \rho_0 \theta_* \frac{1 - \varepsilon_*}{\alpha_1^2} \left[(1 - \varepsilon_*) (\ddot{S}^2 S^2 + \ddot{S} \dot{S} S^2) + 3 \ddot{S} \dot{S}^2 S + \dot{S}^4 \right]$$

Where ε_* - the critical value of the deformation, θ_* - the value of the deformation at the intersection of the straight line $P(t) = k\varepsilon$ with the axis $O\varepsilon$, $\varepsilon_* > \theta_*$, k - some constant, the tangent of the angle of inclination. ρ_0 - the initial density of the soil and - the α_1 velocity of propagation of elastic waves in the pressed soil.[5]

Formulation of the problem

As you know, dropping liquids, such as oil and water, which are most often found in field and domestic practice, are weakly compressible substances. This means that their relative change in volume is only a fraction of a percent. However, when gas bubbles dissolved in the fluid are released, which can change their volume many times over, being, as it were, small springs; as well as the elastic properties of the rock itself - all of the above together cannot negligibly affect the processes of fluid filtration [6, 7]. Let us assume that at one of the points of the reservoir there is a sharp increase in pressure with some value Δp . In view, as mentioned earlier, of the elastic properties of the formation and inertia; the effect of one volume of fluid under pressure on a still undisturbed volume of fluid takes some time. Thus, there is a gradual increase in pressure, layer by layer, which can be interpreted as an oncoming pressure wave. Let's try to consider the pattern of pressure distribution in the reservoir during the propagation of this oncoming pressure wave. Having carried out this study, we obtain characteristic graphs.

MATERIAL AND METHODS

Propagation of a pressure wave in a reservoir in the case of a linear filtration law.

In the first case, we assume that the process of fluid filtration occurs according to a linear law - Darcy's law. This is true at low fluid flow velocities, and, naturally, there can be no talk of turbulences or cavitation breaks. Let us write the continuity equation and the equation of Darcy's law [8], respectively:

$$\frac{\partial(m\rho)}{\partial t} + \text{div}(\rho v) = 0; v = \frac{K_f}{\gamma} \frac{\Delta P}{\Delta L} = -\frac{k}{\mu} \text{grad}(p) \quad (1)$$

Here: m - porosity, ρ - density, v - filtration rate, t - time, K_f - filtration coefficient, γ - specific gravity of the liquid, ΔP - pressure change, ΔL - distance interval, k - permeability coefficient, μ - dynamic viscosity.

Substituting Darcy's law into the continuity equation, we get [6]:

$$\frac{\partial(m(p)\rho(p))}{\partial t} - \text{div}\left(\rho(p) \frac{k(p)}{\mu(p)} \text{grad}(p)\right) = 0 \quad (2)$$

Where each of the coefficients depends on the pressure p . Then, we can write:

$$\left(\rho \frac{\partial m}{\partial p} + m \frac{\partial \rho}{\partial p}\right) \frac{\partial p}{\partial t} - \left[\left(\frac{k}{\mu} \frac{\partial \rho}{\partial p} + \frac{\rho}{\mu} \frac{\partial k}{\partial p} + \rho k \frac{\partial(1/\mu)}{\partial p}\right) [\text{grad}(p)]^2 + \frac{\rho k}{\mu} \text{div}(\text{grad}(p))\right] = 0 \quad (3)$$

We accept the perturbations as small, and then we can say that the coefficients k , μ , ρ and m will be taken as their initial unperturbed values k_0 , μ_0 , ρ_0 and m_0 , and the pressure derivatives will be represented as the corresponding compressibility coefficients.

$$\frac{1}{m} \frac{\partial m}{\partial p} = \beta_m; \frac{1}{\rho} \frac{\partial \rho}{\partial p} = \beta_\rho; \mu \frac{\partial(1/\mu)}{\partial p} = \beta_\mu; \frac{1}{k} \frac{\partial k}{\partial p} = \beta_k. \quad (4)$$

Then we can completely neglect part of the expression as a non-linear term

$$\left(\frac{\partial \rho}{\partial p} + \frac{\partial k}{\partial p} + \frac{\partial(1/\mu)}{\partial p}\right) [\text{grad}(p)]^2 = 0 \quad (5)$$

Thus, we rewrite equation (3):

$$\rho_0 m_0 (\beta_m + \beta_\rho) \frac{\partial p}{\partial t} - \frac{\rho_0 k_0}{\mu l_0} [\text{div}(\text{grad}(p))] = 0 \quad (6)$$



Denote:

$$\frac{k_0}{m_0 \mu_0 (\beta_m + \beta_\rho)} = \chi \quad (7)$$

χ - coefficient of piezoconductivity. The piezoconductivity coefficient simultaneously takes into account the compressibility of the liquid and the porous skeleton. This coefficient acts as a kind of measure of the inertia of the impact on the formation or, in other words, characterizes the rate of pressure propagation in an elastic porous medium. For example, when gas bubbles dissolved in the fluid are released, the compressibility of the system sharply increases and the piezoconductivity coefficient, respectively, also sharply decreases. Let us now consider the one-dimensional problem of the motion of a plane pressure wave in a semi-infinite porous reservoir. Let us assume that the initial pressure of the fluid in the reservoir is constant, and at the zero moment of time the pressure at the boundary of the reservoir jumps up by a constant value Δp and is then maintained at this level all the time. Considering for simplicity the initial pressure to be zero and directing the Ox axis along the reservoir, we obtain the problem

$$\frac{\partial p}{\partial t} = \chi \frac{\partial^2 p}{\partial x^2}; p(x, 0) = 0; p(0, t) = \Delta p; \lim_{x \rightarrow \infty} p(x, t) = 0 \quad (8)$$

According to Leontiev, the solution of the problem is self-similar and, in accordance with the π - theorem, reduces (8) to an ordinary differential equation, the solution of which has the form [8]:

$$p(x, t) = \Delta p \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{t\chi}} \right) \right] \quad (9)$$

Which gives the following pressure distribution graphs (Figure 1-a, b, c, d, e), where a slow increase in pressure over time is obvious for each regular point in space. Here, the approximate values of the coefficients for dolomite rock and medium-viscosity oil were taken [7, 9, 10].

Table 1.

Values of coefficients for the reservoir

Piezoconductivity coefficient, χ , m^2/s	The highest permeability coefficient for dolomite rock, k , m^2	Average density of dolomite rock, m , %	Medium-viscosity oil, μ , Pa s	Pressure jump, Δp , MPa	Compressibility coefficient β_m , m^2/N	Compressibility coefficient β_ρ , m^2/N
11.52023346	1.48035×10^{-10}	0.88	0.005	10	1.1×10^{-9}	1.6×10^{-9}

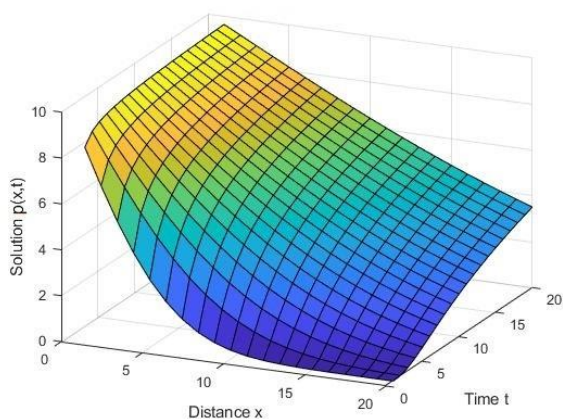


Figure 1 - a .

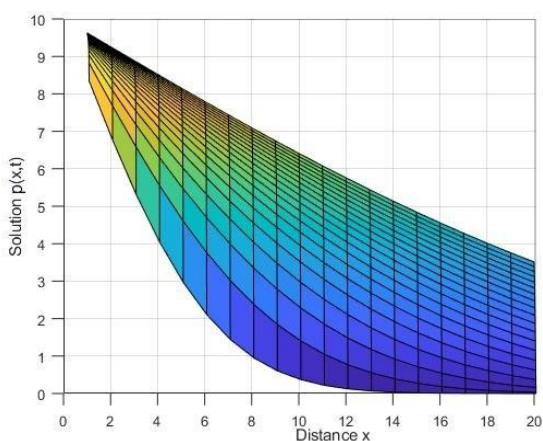


Figure 1 - b.

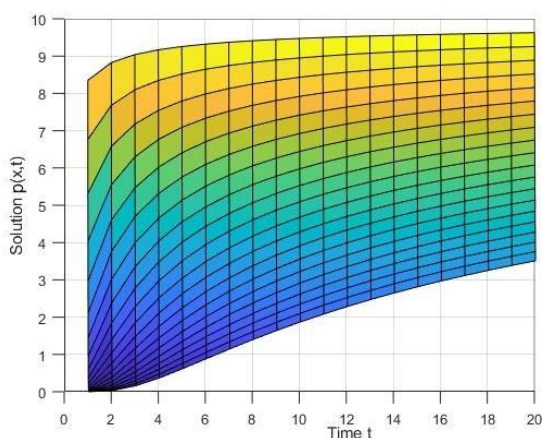


Figure 1 - c.

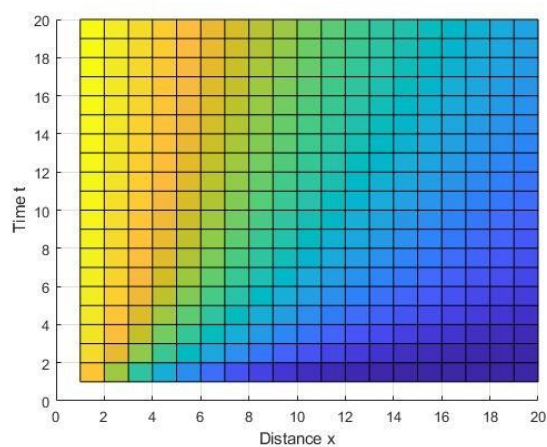


Figure 1 - d.

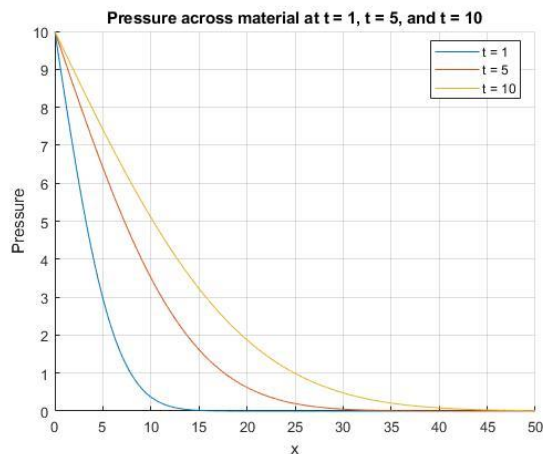


Figure 1- e.

Figure 1. Distribution of pressure in coordinates and time when applying the linear Darcy filtration law.

Propagation of a pressure wave in a reservoir in the case of a nonlinear filtration law.
Let's represent the nonlinear filtering law in the following form [8]:



$$\frac{\Delta P}{\Delta L} = \frac{\mu}{k} v + \frac{\rho}{k_p} v^2 \quad (10)$$

This two-term formula is known as a form of the non-linear Forchheimer-Dupuy filtration law. Here, under k_p - denotes the permeability coefficient of a weighty liquid or, in other words, the coefficient of macro-wideness. Substituting the roots of this expression into the continuity equation, we get:

$$\frac{\partial(m(p)\rho(p))}{\partial t} = \text{div} \left(\frac{\rho(p) \left(-\frac{\mu(p)}{k(p)} \pm \sqrt{\frac{\mu^2(p)}{k^2(p)} + 4 \frac{\rho(p)}{k_p(p)} \text{grad}(p)} \right)}{2 \frac{\rho(p)}{k_p(p)}} \right) \quad (11)$$

Assuming solutions for prostate $[\text{grad}(p)]^2$ and $1/\sqrt{D} \approx 0$, where D is the root expression in (11), after similar transformations we get:

$$\frac{\partial p}{\partial t} = \frac{2\chi}{k_p(p)} \frac{\partial^2 p}{\partial x^2} + \frac{\mu}{2k} (\beta_\mu + \beta_k) (2 - k_p) \frac{\partial p}{\partial x} \quad (12)$$

If we accept the following values for the coefficients (Table 2.):

Table 2.

Coefficients of weighty liquid permeability k_p and compressibilities β_μ, β_k of the reservoir.

Weighty liquid permeability coefficient, k_p , m^2	Compressibility coefficient β_μ , m^2/N	Compressibility coefficient β_k , m^2/N
$1.2167 \cdot 10^{-5}$	$5 \cdot 10^{-3}$	$1.46 \cdot 10^{-10}$

Then we get the following graphs of the form (Figure 2- a, b, c, d, e):

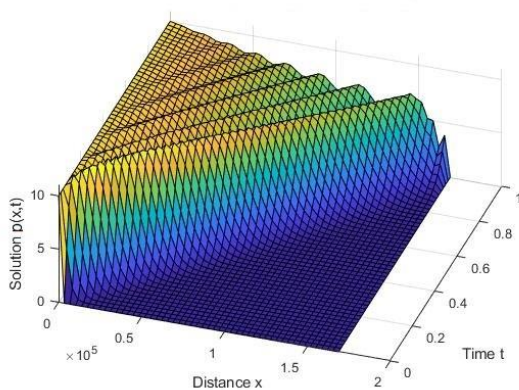


Figure 2- a .

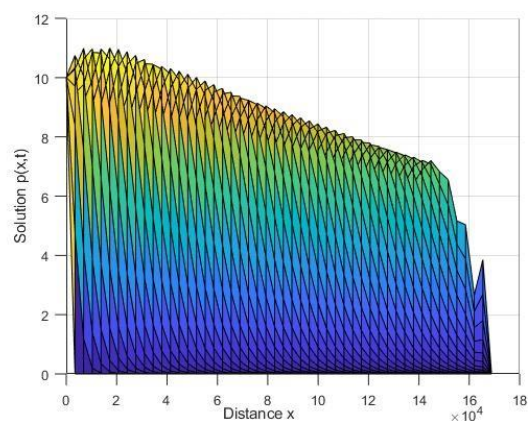


Figure 2- b.

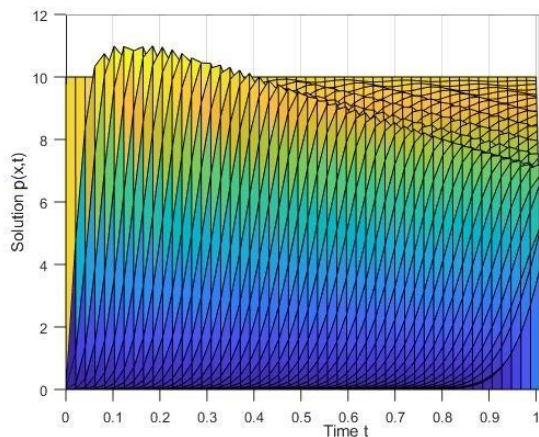


Figure 2- c .

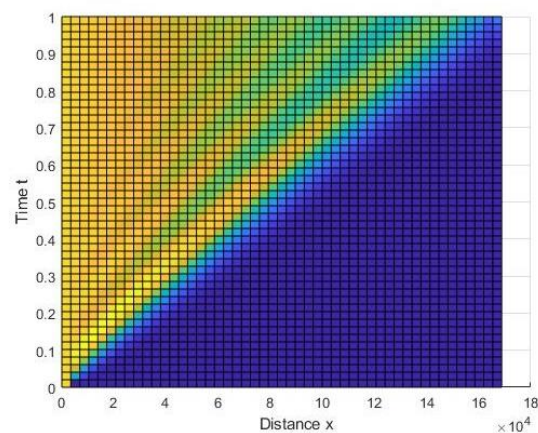


Figure 2- d.

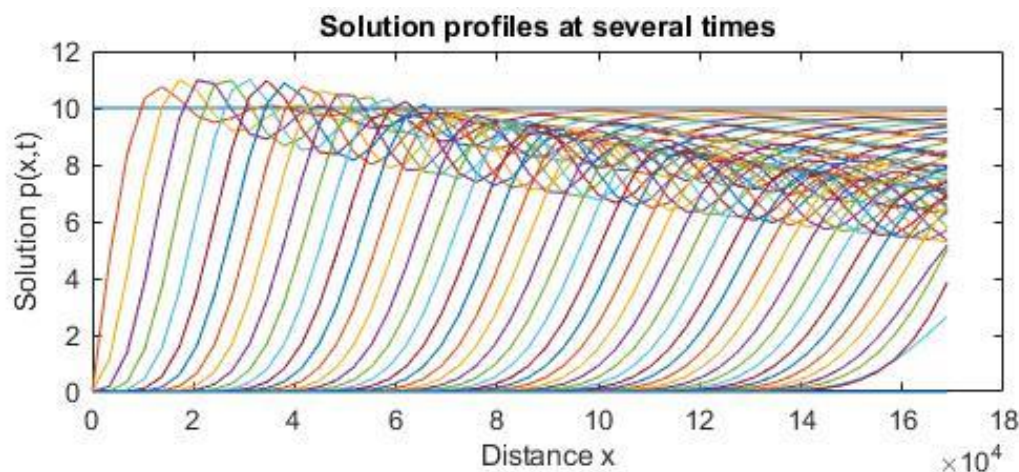


Figure 2- e .

Figure 2. Distribution of pressure in coordinates and time when applying a nonlinear filtration law of the Forchheimer-Dupuy type

CONCLUSION

In contrast to the case of applying the linear filtration law, when applying the non-linear filtration law, the pressure distribution graphs in the reservoir show a difference. Namely: if in the first case, it is clearly seen that, starting from a certain point, the pressure in the reservoir steadily increases layer by layer to the value Δp (Figure 1: a, b, c, d). Then in the second case, there are the following features when taking into account the effects of nonlinear parameters. Therefore, here the pressure fluctuates with damping around the value Δp while tending to this value. In this case, the increase in pressure at the initial and nearby points occurs with a slight delay. Moreover, at the same time, it turns out to be slightly larger than the true value Δp . (Figure 2: a, b, c, d). And if in the first case with a distance there is a sharp increase in pressure to Δp followed by a smooth decrease to zero (for each moment of time, when a pressure wave passes through this point in space); (Figure 1:e) . That is, in the second case, the pressure gradually increases and decreases oscillatingly for a given moment of time t . And the farther the coordinate is, at the moment, x the smaller the amplitude, but the longer the decay time, the more the pressure value will fluctuate until it reaches Δp (Figure 2:e).



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QEYRİ -XƏTTİ SÜZÜLMƏ QANUNU ŞƏRAİTİNDƏ MƏSAMƏLİ MÜHİTDƏ TƏZYİQ DALĞASININ YAYILMASI

B.N. Sultanov

Təzyiq dalğasının layda yayılması həm süxurun özünün, həm də mayenin sıxılma qabiliyyətini nəzərə almaqla nəzərə alınır. Bu zaman iki hal nəzərdən keçirilir: xətti filtrasiya qanunu - Darsi qanunu, və Forchheimer şəklində ümumiləşdirilmiş filtrasiya qanunu. Təzyiqdə kəskin artımla şok dalğası hərəkəti prosesində təzyiqin zaman və məkanda paylanması qrafikləri alınmışdır.

Açar sözlər: *zərbə dalğası, süxurların deformasiyası, qeyri-xətti Forchheimer qanunu*

РАСПРОСТРАНЕНИЕ ВОЛНЫ ДАВЛЕНИЯ В ПОРИСТОЙ СРЕДЕ В СЛУЧАЕ НЕЛИНЕЙНОГО ЗАКОНА ФИЛЬТРАЦИИ

Б.Н. Султанов

Рассматривается распространение волны давления в пласте при учете сжимаемости как самой горной породы, так и флюида. При этом рассматривается два случая: линейный закон фильтрации – закон Дарси и обобщенный закон фильтрации в виде Форхгеймера. Получены графики распределения давления по времени и в пространстве в процессе ударно-волнового воздействия с резким повышением давления.

Ключевые слова: *ударная волна, деформация горных пород, нелинейный закон Форхгеймера.*