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PROBLEMS OF MATHEMATICAL MODELING AND CONTROL OF QUADCOPTER MOTION

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This article describes the general principles of flight and control of a quadcopter, on the basis of which a mathematical model and principles of control of quadcopter motion are developed taking into account the quadcopter dynamics.

Keywords: *unmanned aerial vehicle, quadcopter, mathematical model, motion control*

Introduction

The state of the art in computing and information technology allows using mathematical modeling as a tool for solving problems of control of complex technical systems at a fundamentally new level. The use of modern mathematical application software packages makes it possible to carry out multifaceted studies with high accuracy and with minimal costs of resources of these technical systems.

An example of these complex systems is unmanned aerial vehicles (UAVs), in particular quadcopters, which have recently become increasingly popular as affordable and relatively inexpensive technical means of remote data collection, environmental monitoring, delivery of small-sized cargo, and a number of tasks.

Recent development of unmanned aerial vehicles, including quadcopters, makes it possible to use them in various fields of human activity. Light weight, small size, maneuverability, ease of control is the main advantages of quadcopters, which allow them to be used in many industries, including in the military field. For this reason, there is a need for a mathematical model that it could describe control of an UAV. The difficulties lies in the fact that a quadcopter has six degrees of freedom, while we can control only four parameters: the rotational speeds of the propellers. The process of controlling the quadcopter flight dynamics should be carried out on the basis of an adequate mathematical model [1].

The significant place in engineering practice today is occupied by mathematical problems [2, 3, 4], which, in one form to another, can be associated with the control of mechanical systems [5]. Note that the problem of mathematical modeling and control of quadcopter motion is topical among the various problems of stability and control of UAV motion [6, 7].

A quadcopter is a highly maneuverable aircraft that has poor stability, since its dynamics are highly susceptible to perturbations due to its fairy small mass [8]. A quadcopter control system should in principle solve the problems of angular and spatial stabilization, and the rise to a given altitude (takeoff), and ensure landing, hovering and flight along a given trajectory [9, 10]. In the general case, given these restrictions, quite high requirements are imposed on a quadcopter control system in terms of accuracy and speed.

Many studies devoted to quadcopter motion modeling [6, 7, 9, 11, 12, 13, 14], where various versions of motion equations with various automatic control and stabilization systems are proposed.

For instance, in [6], a dynamic model of a four-rotor rotorcraft, i.e., quadcopter, is constructed using the Lagrange method. In [7], an algorithm is presented for quadcopter control synthesis based on a mathematical model of aircraft motion. Two options for controlling the horizontal movement of the aircraft are considered here, one with specified end point and the other with specified cruise speed.

In this article, we propose a simulation of a system of control and stabilization of quadcopter motion. Using the results of works [6, 7], a simulation of a quadcopter motion control system is presented. The problem involves synthesizing a mode of quadcopter motion control such that would allow the quadcopter to move from a certain starting point to a specified point in space and with specified angles of yaw, pitch and roll.

To solve this problem, initially, as in [7], an algorithm for controlling the flight altitude along the *axis z and the yaw angle ψ is proposed. Next, we consider the control of movement along the axis y and the roll angle φ. Then a system for controlling the movement of the quadcopter along the axis x and the pitch angle θ is developed. And finally, by combining z, y and x, we obtain the flight path of the quadcopter.

Calculations are carried out on the basis of the control algorithm, and the calculation results are illustrated with specific examples.

Problem statement

Consider a quadcopter (Fig. 1) with known physical parameters, the motion of which can be controlled by changing the speed of the propellers.

Fig. 1. Quadcopter

The aircraft moves relative to an Earth-fixed inertial frame of reference and specified by the coordinate axes Ox, Oy and Oz that are perpendicular to one another, with the axis Oz directed in opposition to the gravity vector. The problem involves making the quadcopter move from the starting point (x_0, y_0, z_0) with initial yaw angle ψ_0, roll angle φ_0 and pitch angle θ_0 to a specified point (x_d, y_d, z_d) with specified yaw angle ψ_d, roll angle φ_d and pitch angle θ_d. The diagram of such a quadcopter is shown in Fig. 2 (see [6, 7]).

As noted in [7], let $\xi = \left[x \;\; y \;\; z\right]'$ be the radius vector of the quadcopter center of mass, ψ, θ, φ are the yaw, pitch and roll angles respectively. f_i is the lifting force produced by the i-th motor M_i $(i = \overline{1,4})$. Here and further the prime denotes transposing.

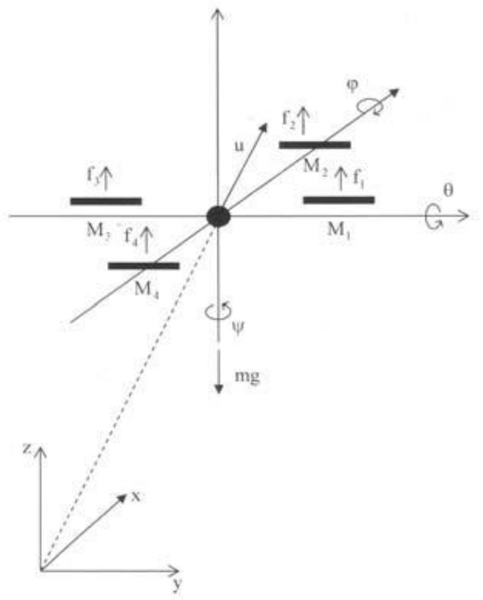

Fig. 2. The diagram of a quadcopter

In accordance with [6, 7], the motion of this system is described by the following equations:

$$m\ddot{x} = -u\sin\theta, \tag{1}$$

$$m\ddot{y} = u\cos\theta\sin\varphi, \tag{2}$$

$$m\ddot{z} = u\cos\theta\cos\varphi - mg, \tag{3}$$

$$\ddot{\psi} = \tilde{\tau}_{\psi}, \tag{4}$$

$$\ddot{\theta} = \tilde{\tau}_\theta, \tag{5}$$

$$\ddot{\phi} = \tilde{\tau}_\varphi. \tag{6}$$

In equations (1) – (6), m is the mass of the quadcopter, $g = 9{,}8$ m/s is the gravity acceleration, u, as well as $\tilde{\tau}_\psi$, $\tilde{\tau}_\theta$, $\tilde{\tau}_\varphi$ are the control inputs, which are the functions of f_i. In [6], the control input u is used for the quadcopter altitude control, and the control input $\tilde{\tau}_\psi$ allows stabilizing the yaw angle. In accordance with [7], the control inputs $\tilde{\tau}_\theta$ and $\tilde{\tau}_\varphi$ are used for control of the pitch θ and roll φ angles, as well as for control of the quadcopter motion along the axes x and y.

As shown in [6, 7], if we suppose that $\cos\theta\cos\varphi \neq 0$, control of the quadcopter flight altitude is defined by the following relation:

$$u = (r_1 + mg)\frac{1}{\cos\theta\cos\varphi}, \tag{7}$$

where

$$r_1 = -a_{z_1}\dot{z} - a_{z_2}(z - z_d). \tag{8}$$

In formula (8), a_{z_1}, a_{z_2} are positive constants, and z_d is the desired flight altitude. Taking into account (7) and (8), from (3) we have

$$m\ddot{z} = u\cos\theta\cos\varphi - mg = r_1 = -a_{z_1}\dot{z} - a_{z_2}(z - z_d).$$

Similarly, for the control of the yaw angle, we have

$$\tilde{\tau}_\psi = -a_{\psi_1}\dot{\psi} - a_{\psi_2}(\psi - \psi_d) \tag{9}$$

Then at $\cos\theta\cos\varphi \neq 0$ and in accordance with (7) – (9), from formulas (1)-(4) we have:

$$m\ddot{x} = -(r_1 + mg)\frac{\tan\theta}{\cos\varphi}, \tag{10}$$

$$m\ddot{y} = (r_1 + mg)\tan\varphi, \tag{11}$$

$$\ddot{z} = \frac{1}{m}\left(a_{z_1}\dot{z} - a_{z_2}(z - z_d)\right), \tag{12}$$

$$\ddot{\psi} = -a_{\psi_1}\dot{\psi} - a_{\psi_2}(\psi - \psi_d). \tag{13}$$

In equations (12), (13), the unknown coefficients a_{z_1}, a_{z_2}, a_{ψ_1}, a_{ψ_2} must be chosen from the asymptotic stability conditions in the vertical direction and along the yaw angle, which, in turn, satisfies the condition $z \to z_d$, $\psi \to \psi_d$.

After ensuring control of the flight altitude and the yaw angle, we can proceed to the control of the horizontal movement and the roll and pitch angles.

As shown in [6], in the case of time tending to infinity, from (8) and (12) we have $r_1 \to 0$. Then, for large values of time T, the values of r_1 and ψ will be sufficiently small, and from (10) and (11) we have

$$\ddot{x} = -g\frac{\tan\theta}{\cos\varphi}, \tag{14}$$

$$\ddot{y} = g\tan\varphi. \tag{15}$$

It should be noted that in [6, 7], a nonlinear algorithm [15] is used for stabilization of the coordinates x, y, taking into account (14), (15).

Further, considering the angles θ, φ small in (14) and (15), as in [6, 7], and taking in account equations (5), (6), we have the following relations defining the variation of these coordinates:

$$\ddot{y} = g\varphi, \tag{16}$$

$$\ddot{\varphi} = \tilde{\tau}_\varphi, \tag{17}$$

$$\ddot{x} = -g\theta, \tag{18}$$

$$\ddot{\theta} = \tilde{\tau}_\theta. \tag{19}$$

Thus, here, as in [7], we will propose a linear algorithm for stabilizing the horizontal movement of the quadcopter. First, we will consider the control of the coordinates (φ, y). Denoting

$$p_1 = [(y - y_d), \frac{d(y - y_d)}{dt}, (\varphi - \varphi_d), \frac{d(\varphi - \varphi_d)}{dt}]',$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}',$$

$$u_1 = \tilde{\tau}_{\varphi},$$

from subsystem (18)-(19), we get

$$\begin{aligned} \dot{p}_1 &= A_1 p_1 + B_1 u_1, \\ p_1(0) &= p_1^0. \end{aligned} \tag{20}$$

To synthesize the optimal linear controller of problem (20), we will use the following performance index

$$J_1 = \int_0^\infty (p_1' Q_1 p_1 + u_1' R_1 u_1) dt, \tag{21}$$

where Q_1 and R_1 are matrices satisfying solutions of the synthesis problem for control of the movement along the axis y and the angle φ. As a result of solving the synthesis problem and applying the standard LQR synthesis procedure, we have the state feedback gains $u_1 = -K_y p_1$ in the form of $K_y = [a_{y_1}, a_{y_2}, a_{\varphi_1}, a_{\varphi_2}]$, thus providing for $y \to y_d, \varphi \to \varphi_d$ at $t \to \infty$.

We will now consider the control of the coordinates (θ, x). In this case, denoting

$$p_1 = [(x - x_d), \frac{d(x - x_d)}{dt}, (\theta - \theta_d), \frac{d(\theta - \theta_d)}{dt}]',$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}',$$

$$u_2 = \tilde{\tau}_{\theta},$$

from subsystem (18)-(19), we get

$$\begin{aligned} \dot{p}_2 &= A_2 p_2 + B_2 u_2, \\ p_2(0) &= p_2^0. \end{aligned} \tag{22}$$

To synthesize the optimal linear controller of problem (22), in this case we will use the following performance index

$$J_2 = \int_0^\infty (p_2' Q_2 p_2 + u_2' R_2 u_2) dt, \tag{23}$$

where Q_2 and R_2 are matrices satisfying solutions of the synthesis problem for control of the movement along the axis x and the angle θ. As a result of solving the synthesis problem and applying the standard LQR synthesis procedure, we have the state feedback gains $u_2 = -K_x p_2$ in the form of $K_x = [a_{x_1}, a_{x_2}, a_{\theta_1}, a_{\theta_2}]$, thus providing for $x \to x_d, \theta \to \theta_d$ at $t \to \infty$.

To solve systems (12) and (13), we proceed as follows. First, we consider (12). Suppose that

$$p_3 = [(z - z_d), \frac{d(z - z_d)}{dt}]',$$

Denote

$$A_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{24}$$

Then from (12) we get

$$\begin{aligned} \dot{p}_3 &= A_3 p_3 + B_3 u_3, \\ p_3(0) &= p_3^0. \end{aligned} \tag{25}$$

Further, in this case we will use the following performance index

$$J_3 = \int_0^\infty (p_3' Q_3 p_3 + u_3' R_3 u_3) dt, \tag{26}$$

where Q_3 and R_3 are matrices satisfying solutions of the synthesis problem for control of the movement along the axis z. As a result of solving the synthesis problem and applying the standard LQR synthesis procedure, we have the state feedback gains $u_3 = -K_z p_3$ in the form of $K_z = [a_{z_1}, a_{z_2}]$ or $u_3 = r_1 = -a_{z_1}\dot{z} - a_{z_2}(z - z_d)$, thus providing for $z \to z_d$ at $t \to \infty$.

In a similar manner, we can solve (13).

Then, substituting $u_1 = -K_y p_1$, $u_2 = -K_x p_2$ and $u_3 = -K_z p_3$ in (20), (22) and (25), respectively, and solving these Cauchy problems, we get the solutions that are the coordinates of the quadcopter flight path.

Thus, as the result of quadcopter motion control synthesis, we have ($x(t), y(t), z(t)$), for which $\theta \to \theta_d$, $x \to x_d$, $y \to y_d, \varphi \to \varphi_d$, $\psi \to \psi_d$, $z \to z_d$ at $t \to \infty$.

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KVADROKOPTERİN HƏRƏKƏTİNİN VƏ İDARƏ OLUNMASININ RİYAZİ MODELLƏŞDİRİLMƏSİ PROBLEMLƏRİ

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Bu işdə kvadrokopterin uçuşunun və idarə olunmasının ümumi prinsipləri təsvir edilmişdir ki, bunun da əsasında kvadrokopterin hərəkətinin riyazi modeli, habelə kvadrokopterin dinamikası nəzərə alınmaqla idarə olunması prinsipləri işlənmişdir.

Açar sözlər: *pilotsuz uçuş aparatı, kvadrokopter, riyazi model, hərəkətin idarə olunması*

ПРОБЛЕМЫ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ ДВИЖЕНИЕМ И УПРАВЛЕНИЯ КВАДРОКОПТЕРА

М.М. Муталлимов, Н.И. Велиева, А.М.Аббасов, Ф.А. Алиев

В данной статье описаны общие принципы полета и управления квадрокоптером, на основе которых разрабатывается математическая модель движением квадрокоптера, а также принципы управления с учетом динамики квадрокоптера.

Ключевые слова: *беспилотный летательный аппарат, квадрокоптер, математическая*