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OPTIMAL ROUTE SELECTION PROBLEM IN OPERATIONAL AND ORGANIZATIONAL CONTROL

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Three equivalent formulation forms of the optimal search problem are considered. To solve the problem, the dynamic programming method, the branch-and-bound method, the "nearest neighbor" method, as well as heuristic methods were used. To increase the efficiency of the branch and bound method, a number of properties of the Hamiltonian contour are used.

Keywords: *operational control, optimal route selection problem, dynamic programming, branch and bound method, Hamiltonian contour*

Formulation of the problem

Suppose that there are n points (regions) on a geographical map, each of which may contain an object of interest to us (a lost expedition, a missing ship, etc.), and there is a search tool (an airplane, a rescue ship, etc.), which is able to examine all n points. The distances $a_{i,j}$ between any points i and $j(i, j = \overline{1,n})$ are specified. It is required to select a search route that starts and ends at some point i_1, and successively covers all the other $(n-1)$ points, passing through them only once and having the shortest length.

The optimal route selection problem is also called the optimal search problem or an assignment problem [1], or the traveling salesman problem(TSP) [2]. The last term is the most common, and we will use it further.

There are three equivalent formulations of this problem.

1. n cities and distances between them are specified. A traveling salesman leaving a city i_1 must visit each of the $(n-1)$ other cities only once and return to the initial city, traveling the minimum distance.

2. A simple graph $G = (J, U)$ is specified, where J is the set of vertices and U is the set of arcs (edges) of the graph. It is required to determine the shortest Hamiltonian circuit possible. (A Hamiltonian circuit in a graph is a path that visits each vertex of the graph G only once; an Eulerian circuit is a path that visits each arc of the graph only once).

3. A square weight matrix A (of the n-th order) is specified. It is required to determine the permutation

$$i_1, i_2, i_3, \ldots, i_{n-1}, i_n, \quad i_j \neq i_q \text{ when } j \neq q,$$

which would minimize the functional

$$a_{i_1,i_n} + \sum_{j=1}^{n-1} a_{i_j,i_{j+1}}.$$

There are practically important cases when the elements of the matrix $a_{i,j}$ are non-stationary; $a_{i,j} = a_{i,j}(t)$, which indicates "moving points" $i_1, \ldots, i_n$ in space. In particular, work [3] is devoted to the solution of such problems, which is called the "m traveling salesmen" problem [4].

Finally, it may turn out that $a_{i,j} \neq a_{j,i}$, i.e., a path from i to j is not equivalent to a reverse path from j to i. Such a problem is called the asymmetric TSP and requires consideration of the complete matrix A, in contrast to the symmetric problem, where, since $a_{i,j} = a_{j,i}$, it is sufficient to consider the triangular matrix obtained from the matrix A.

In what follows, we limit ourselves to the consideration of the so-called classical traveling salesman problem, for which: 1) $a_{i,j}$ are stationary, 2) $a_{i,j} = a_{j,i}$; 3) $m = 1$, since the methods for solving this problem underlie all modifications of the TSP and the difficulty of solving the classical problem is sufficient to demonstrate all approaches to its solution.

It may seem that the TSP is very close to the assignment problem. Indeed, any Hamiltonian circuit in the graph

$$\Gamma = \{i_1 \to i_2 \to i_3 \to \ldots \to i_{n-1} \to i_1\}$$

can be written as a set of arcs

$$D = \{i_1\hat{}i_2,\ i_2\hat{}i_3, \ldots,\ i_n\hat{}i_1\},$$

characterized by the elements of the matrix A

$$a = \{a_{i_1,i_2}, a_{i_2,i_3}, \ldots, a_{i_n,i_1}\}.$$

Analyzing the last set, we can see that in the complete matrix A, to solve the traveling salesman problem, we must select one element from each column and each row (diagonal elements cannot be selected). Therefore, denoting the choice of the element $a_{i,j}$ by the symbol $x_{i,j} = 1$, we arrive at the following problem:

$$E = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{i,j}x_{i,j} \to \min;$$

$$\sum_{i=1}^{n} x_{i,j} = 1; \quad \sum_{j=1}^{n} x_{i,j} = 1.$$

This problem, up to notation, agrees with the assignment problem, but does not formalize all the conditions of the TSP. Solving the traveling salesman problem in does not at all guarantee that the resulting circuit will turn out to be closed. To ensure the closure conditions, it is necessary to impose additional constraints [5], [6]. This significantly complicates the calculation procedure and limits the possibilities of the solution to the values $n \le 10$.

The dynamic programming method

Using the dynamic programming method for solving the TSP was first proposed by R. Bellman [7], [8] and, independently, by M. Held and R. Karp [9]. Later this method was used by many authors [2, 7, 10, 11]. Suppose that n vertices are specified. We select any vertex among them and call it the initial and final vertices. Since the Hamiltonian circuit is closed, the initial and final vertices can be selected arbitrarily. Let us designate it as number 1. The process of solving the problem will represent an $(n-1)$-step process of considering an increasing number of vertices, starting from 1 and ending with n.

Suppose that the Bellman function, written in the form $\omega_k(i_1 \to i_2, i_3, \ldots, i_{k-1}, 1)$, gives the length of the minimum path that starts at vertex i_1, ends at vertex 1 and goes through vertices $i_2, i_3, \ldots, i_{k-1}$, covering k vertices. Obviously, since $\omega_1 = 0$, the solution process starts in the second step with the calculation of $\omega_2(i_1 \to 1)$ for all admissible $i_1 = \overline{2, n}$, and ends in the last step with obtaining $\omega_n(1 \to i_2, i_3, \ldots, i_{n-1}, 1)$, giving the optimal solution to the TSP.

In the third step, since there exist on more than two circuits connecting three vertices, we get

$$\omega_3(i_1 \to i_2, 1) = a_{i_1,i_2} + a_{i_2,1};$$
$$\omega_3(i_2 \to i_1, 1) = a_{i_2,i_1} + a_{i_1,1}.$$

The fourth step will contain the minimization procedure corresponding to the functional equation

$$\omega_4(i_1 \to i_2, i_3, 1) = \\ = \min_{\{i_2\}}\left[a_{i_1,i_2} + \omega_3(i_2 \to i_3, 1)\right].$$

Obviously, the number of possible values of ω_4 for the fixed vertex i_1 will be the number of combinations $\binom{n-2}{2}$, and $(n-1)\times\binom{n-2}{2}$ in total, for ω_5 we get a total of $(n-1)\times\binom{n-2}{3}$ options, etc.

By analogy, we write functional equations for an arbitrary k-th step and the last n-th step:

$$\omega_k(i_1 \to i_2,\ldots,i_{k-1},1) = \\ = \min_{\{i_2\}} \left[a_{i_1,i_2} + \right. \\ \left. + \omega_{k-1}(i_2 \to i_3,\ldots,i_{k-1},1)\right] \quad (1)$$

$$\omega_n(1 \to i_2,\ldots,i_n,1) = \\ = \min_{\{i_2\}} \left[a_{1,i_2} + \right. \\ \left. + \omega_{n-1}(i_2 \to i_3,\ldots,i_n,1)\right] \quad (2)$$

The largest amount of computation will correspond to steps with numbers close to $\frac{n-2}{2}+2$. For instance, for an even n, for a step with number $k=\left(\frac{n-2}{2}+2\right)$, we will need to analyze and store in the computer memory the largest number $(n-1)\binom{n-2}{(n-2)/2}$ of values of ω. This fact mainly determines the complexity of the dynamic programming method.

The branch and bound method

The main method for solving TSP that ensures finding the optimal solution is the branch and bound method, first proposed in [1] and later widely used both in the authors' version and in its various modifications. Using modern computers, the branch and bound method can be used to solve TSPs for $n \le 35 \div 40$.

The main idea of the method is that, first, we construct a lower bound for the lengths of routes for the entire set of Hamiltonian circuits, then the entire set of circuits τ^0 is divided into two subsets so that the first subset $\tau^1_{i,j}$ consists of Hamiltonian circuits containing an arc $\widehat{i,j}$, and another subset of Hamiltonian circuits $\bar{\tau}^1_{i,j}$ does not contain this arc.

Let us now focus on the method for determining the lower bounds of subsets and methods for the best division of subsets. The determination of the lower bounds is based on the following theorem:

Theorem 1. If a positive or negative number α is added to all elements of the i-th row (or j-th column) of the matrix A of the TSP, then the problem remains equivalent to the previous one, and the length of any Hamiltonian circuit changes by the specified constant value α

Based on Theorem 1, we subtract from each row the number α_i equal to the minimum element of this row

$$\alpha^1_{i,j} = \alpha_{i,j} - \alpha_i, \\ i=\overline{1,n};\ \alpha_i = \min_j a_{i,j},$$

and obtain the matrix $\tilde{A}^{(1)}$ called a row-reduced matrix. The matrix $\tilde{A}^{(1)}$ will contain at least one zero in each row i, and all its elements will be non-negative. Then we subtract from each j-th column of the matrix $\tilde{A}^{(1)}$ the number α_i equal to the minimum element of this row:

$$\tilde{\alpha}_{j,i} = \alpha^1_{j,i} - \alpha_j,\ \ j=\overline{1,n},\ \ \alpha_j = \min_i a^1_{j,i}.$$

The matrix $\tilde{A}$ called a row and column-reduced matrix. This matrix will contain at least one zero in each row and each column for non-negative values of the rest of the elements. α will be called the constant of reduction of the matrix A,

$$\alpha = \sum_{i=1}^{n} \alpha_i + \sum_{j=1}^{n} \alpha_j.$$

The value of α will in this case be the lower bound of the entire set of solutions. Indeed, we denote by $d(l)$ the length of any circuit in the problem with the matrix A, and by $d(l_1)$ the length of the same circuit in the problem with the matrix $\tilde{A}$. By virtue of Theorem 1, we can write $d(l) = d(l_1) + \alpha$.

The first subset $\tau^1_{i,j}$ is made up of Hamiltonian circuits containing this arc. Therefore, an a priori inclusion of $\widehat{i,j}$ into the circuit leads to automatic reduction of the matrix $\tilde{A}$ by row i and column j. The resulting matrix $\tilde{A}\left(\tau^1_{i,j}\right)$ describes all Hamiltonian circuits of the set $\tau^1_{i,j}$. By performing a row and column reduction of the matrix $\tilde{A}(\tau_{i,j})$, we obtain the reduction constant $\alpha^1_{i,j}$, which allows us to determine the lower bound $h^1_{i,j}$ of the subset $\tau^1_{i,j}$ by the formula

$$h^1_{i,j} = \alpha + \alpha^1_{i,j}. \tag{3}$$

The second subset $\bar{\tau}^1_{i,j}$ will consist of Hamiltonian circuits not containing the arc $\hat{i,j}$. Therefore, we need to assume $\tilde{\alpha}_{i,j} = \infty$. Carrying out the reduction, we obtain the matrix $\tilde{A}(\bar{\tau}^1_{i,j})$, characterized by the reduction constant $\bar{\alpha}^1_{i,j}$. The lower bound of the subset $\bar{\tau}^{-1}_{i,j}$ is the value

$$\bar{h}^1_{i,j} = \alpha + \bar{\alpha}^1_{i,j}. \tag{4}$$

Instead of (4), we can use another estimate [2]. Let us calculate the "penalty" $p_{i,j}$ for not using the arc $\hat{i,j}$. If $\hat{i,j}$ is not included in the Hamiltonian circuit, then it will necessarily include one element from the i-th row $\tilde{A}(\bar{\tau}^1_{i,j})$ and one element from its j-th column. The smallest penalty will be

$$p_{i,j} = \min_{\beta} \tilde{\alpha}_{i,\beta} + \min_{\gamma} \tilde{\alpha}_{\gamma,j} \tag{5}$$

therefore,

$$\bar{h}^1_{i,j} = \alpha + p_{i,j}. \tag{6}$$

The question remains open about the selection of an arc $\hat{i,j}$ such that the analysis tree is as short as possible. The desired circuit is most likely to belong to those arcs whose lengths in $\tilde{A}(\tau^k_{i,j})$ are equal to zero. The selection of a single arc from all $\hat{i,j}$, for which $\tilde{\alpha}_{i.j} = 0$, is carried out either by reducing the matrix $\tilde{A}(\bar{\tau}^k_{i,j})$, where for the analyzed arc $\hat{i,j}$ we assume $\alpha_{i.j} = \infty$, or by calculating the "penalty" $p_{i.j}$ for not using the arc $\hat{i,j}$.

A good example illustrating the use of the branch and bound method for a 10th order non-symmetric matrix is contained in [12]. Let us look at a simpler example.

The graph of the problem is presented in Fig. 1.

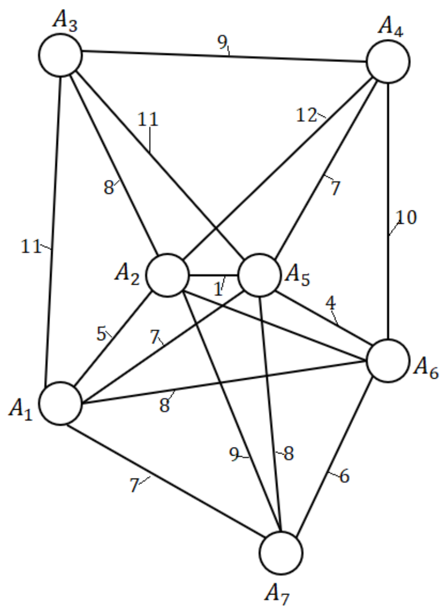

Fig. 1. An example of a graph for the optimal search problem

The numbers near the edges mean their weights, which are also reflected by the matrix A

$$A = \begin{bmatrix} \infty & 5 & 11 & \infty & 7 & 8 & 7 \\ 5 & \infty & 8 & 12 & 1 & 6 & 9 \\ 11 & 8 & \infty & 9 & 11 & \infty & \infty \\ \infty & 12 & 9 & \infty & 7 & 10 & \infty \\ 7 & 1 & 11 & 7 & \infty & 4 & 8 \\ 8 & 6 & \infty & 10 & 4 & \infty & 6 \\ 7 & 9 & \infty & \infty & 8 & 6 & 8 \end{bmatrix} \tag{7}$$

As usual, the rows of A will be numbered in ascending order from top to bottom, columns — in ascending order from left to right. A is a symmetric matrix.

Some properties of the Hamiltonian circuit

To increase the efficiency of the branch and bound method, a number of properties of the Hamiltonian circuit can be used, which reduce the analysis of the subsets $\tau_{i,j}^{k}$.

Let a square weight matrix A of the n-th order be given for a simple (i.e., loop-free) directed graph $G(X,U)$. Let us transform A into its equivalent matrix B, for which the k-th row and k-th column will contain zero elements, except for the element $a_{k,k}=\infty$. To this end, using Theorem 7, it is necessary to subtract the element $a_{i,k}$ from the elements of each i-th row $i=\overline{1,n};\ i\neq k$, and then subtract the element $a_{k,j}$ formed after row operations from the elements of each j-th column $j=\overline{1,n};\ j\neq k$.

Let us select the submatrix $B^{(1)}$ from the matrix B, removing the k-th row and k-th column from B. The submatrix $B^{(1)}$ will describe the subgraph $G^{(1)}$ formed from the graph G by removing the vertex k and all arcs incident with it.

Let us expand the elements of the submatrix $B^{(1)}$ into a sequence in decreasing order of their values

$$b_{i_1,j_1}, b_{i_2,j_2}, b_{i_3,j_3},\dots, b_{i_l,j_l},\dots, b_{i_m,j_m},$$

the number of members of which is $m=n^2-3n+2$.

From this sequence we form a sequence of arcs of the subgraph $G^{(1)}$:

$$L^{(k)}=\{(i_1,\hat{j}_1),(i_2,\hat{j}_2),(i_3,\hat{j}_3),\dots,$$
$$\dots,(i_{l-1},\hat{j}_{l-1}),(i_l,\hat{j}_l),(i_{l+1},\hat{j}_{l+1}),\dots,$$
$$\dots,(i_{m-1},\hat{j}_{m-1}),(i_m,\hat{j}_m)\} \qquad (8)$$

after which we cut off from the end of the sequence $L^{(k)}$ a maximal part of it $L_2^{(k)}$ such that from it is impossible to construct its elements a Hamiltonian circuit in the subgraph. The partition of the sequence $L^{(k)}$ into $L_1^{(k)}$ and $L_2^{(k)}$ can be carried out if there is a condition for the existence of a Hamiltonian circuit in the graph.

Let us introduce the following notation:

$\rho^{+}(i)$ – the number of arcs outgoing from the vertex i in a directed graph;

$\rho^{-}(i)$ – the number of arcs entering the vertex i in a directed graph;

$\rho(i)$ – the number of edges incident with the vertex i in an undirected graph.

Since at present there is no necessary and sufficient condition for the existence of a Hamiltonian circuit in a graph, we will formulate only a sufficient one, which is a corollary of Euler's graph theorem [13].

Condition 1. There cannot exist a Hamiltonian circuit in an undirected graph G if it contains a vertex i such that $\rho(i)<2$.

Condition 2. There cannot exist a Hamiltonian circuit in a directed graph G if it contains a vertex i for which $\rho^{+}(i)=0$ or (and) $\rho^{-}(i)=0$.

We denote the optimal Hamiltonian circuit by

$$H^{*}=\{i_1^{*}\to i_2^{*}\to i_3^{*}\to\dots\to i_n^{*}\to i_1^{*}\}.$$

It can also be written as a sequence of arcs

$$D^{*}=\left\{i_1^{*}\hat{i}_2^{*}\to i_2^{*}\hat{i}_3^{*}\to\dots\to i_{n-1}^{*}\hat{i}_n^{*}\to i_n^{*}\hat{i}_1^{*}\right\}.$$

and as a sequence of pairs of arcs

$$C^{*}=\left\{i_1^{*}\hat{i}_2^{*}\hat{i}_3^{*},\dots,i_{n-2}^{*}\hat{i}_{n-1}^{*}\hat{i}_n^{*},i_{n-1}^{*}\hat{i}_n^{*}\hat{i}_1^{*}\right\}.$$

Now we can formulate the following lemmas concerning the properties of sequence (8).

Lemma 1. The first arc in sequence (8) does not belong to H^{*}.

Lemma 2. If the pair of arcs $\hat{i}_p k\hat{i}_{p+1}$ belongs to H^{*}, then none of the arcs located in (8) to the left of the arc $i_p\hat{i}_{p+1}$ belongs to H^{*}.

Lemma 3. The vertex k in the circuit H^{*}

cannot form the pair of arcs $\hat{i}_\lambda k\hat{i}_{\lambda+1}$, if $i_\lambda \hat{i}_{\lambda+1} \in L_2^{(k)}$

Lemma 4. If an arc then the vertex k cannot form the pair of arcs $(\hat{i}_g k\hat{i}_{g+1}) \in C^*$, then $\hat{i}_g \hat{i}_{g+1}$ in (8) are to the right of $\left(i_p\ \hat{i}_{p+1}\right)$.

Proof of the lemmas. Suppose the circuit $H^* = \{i_1 \to i_2 \to ... i_p \to k \to i_{p+1} \to ... \to i_n \to i_1\}$ was transformed into a circuit H by carrying the vertex k

$H = \{i_1 \to i_2 \to ... i_g \to k \to i_{g+1} \to ... \to i_n \to i_1\}, g \neq p.$

For H^* and H , we will have the following values of the criteria E^* and E . We calculate $\Delta = E - E^*$. Since H^* and H differ only in two areas determined by the location of the vertex k , we obtain

$$\Delta = b_{i_g,k} + b_{k,i_{g+1}} + b_{i_p,i_{p+1}} - \\ - b_{i_p,k} - b_{k,i_{p+1}} - b_{i_g,i_{p+1}}$$

From the conditions for the formation of the matrix B , we have

$$b_{i_g,k} = b_{k,i_{g+1}} = b_{i_p,k} = \\ = b_{k,i_{p+1}} = 0$$

Therefore, $\Delta = b_{i_p,i_{p+1}} - b_{i_g,i_{g+1}}$. If the permutation of the vertex k improves the circuit H^*, then it should be $\Delta < 0$, i.e.,

$$b_{i_g,i_{g+1}} > b_{i_p,i_{p+1}} \qquad (9)$$

Now let us suppose that $i_1 \hat{i}_2 \in D^*$ and is in the first place in (4) i.e., $i_2 = j_1$, and the vertex k forms the pair of arcs $\hat{i}_p k \hat{i}_{p+1}$. Let us form a new circuit $H = \{i_1 \to k \to i_2 \to ... ... \to i_n \to i_1$ and calculate Δ . Since due to the properties of (8) $b_{i_1,i_2} = b_{i_g,i_{g+1}} > b_{i_p,i_{p+1}}$, i.e., (9) is fulfilled, we obtain $E < E^*$. Therefore, the circuit H^* is not optimal. Then we assume that H is optimal, but this circuit does not contain the arc $i_1 \hat{i}_2$ $i_1 \hat{i}_1$ in (8) "broken" vertex k on the pair of arcs $\hat{i}_1 k\hat{i}_2$ Lemma 1 is proved.

It follows that the pair of arcs $\hat{i}_p k\hat{i}_{p+1}$ in the optimal Hamiltonian circuit H^* is formed by the “partition” of the arc $i_p\ \hat{i}_{p+1}$, and $b_{i_p,i_{p+1}} \geq b_{i_\lambda,i_{\lambda+1}}$ for all $i_\lambda \in H^*$. But in sequence (8) the arcs to the left of the arc $\hat{i}_p \hat{i}_{p+1}$ are larger than it. Therefore, if H^* is optimal and $\hat{i}_p k\hat{i}_{p+1} \in H^*$, the arcs lying in (8) to the left of $\hat{i}_p \hat{i}_{p+1}$ are not included in H^*. Lemma 2 is proved.

To prove Lemma 3, let us recall that the circuit H^* is formed in the graph G by introducing the vertex k into the subgraph $G^{(1)}$.

The arc $i_\lambda \hat{i}_{\lambda+1} \in L_2^{(k)}$ cannot be in this subgraph due to the definition of $L_2^{(k)}$. But if $i_\lambda \hat{i}_{\lambda+1}$ is not in $G^{(1)}$, the pair of arcs $\hat{i}_\lambda k\hat{i}_{\lambda+1}$ cannot be formed in the graph G either. Lemma 3 is proved.

Finally, let us consider the case when H^* is such that $i_p \hat{i}_{p+1} \in D^*$ and $\hat{i}_g\ k\ \hat{i}_{g+1} \in C^*$, the arc in sequence (8) being located to the right of $i_p \hat{i}_{p+1}$ Then $b_{i_p,i_{p+1}} > b_{i_g,i_{g+1}}$ and H^* is not optimal. Therefore, the condition $\hat{i}_g k\hat{i}_{g+1} \in C^*$ in the presence of $i_p \hat{i}_{p+1} \in C^*$ and $b_{i_p,i_{p+1}} > b_{i_g,i_{g+1}}$ cannot be fulfilled. Lemma 4 is proved.

The conditions defined by the above lemmas can be used as the basis for constructing a modified branch and bound method.

Conclusions

In order to increase the efficiency of search and reduce the time spent on search, in practice, not one, but several search tools can be used.

If the path from i to j is not equivalent to the return path from j to i, then such a traveling salesman problem requires consideration of the full matrix A, in contrast to the symmetric problem, where, due to the fact that $a_{i,j} = a_{j,i}$, it suffices to confine ourselves to considering the triangular matrix obtained from the matrix A.

When using the dynamic programming method, the process of solving the problem will be an $(n-1)$- step process of considering an increasing number of vertices, starting from 1 and ending with n. However, the advantage of dynamic programming over exhaustive enumeration is that instead of direct comparison $(n-1)!$ variants (for an asymmetric problem), a significantly smaller number of combinations is analyzed, and at all stages of solving the problem $n^2 2^{n-1}$ comparisons are made. Currently, this method allows solving the TSPs for $n \le 17$.

The main method for solving TSPs, which provides finding the optimal solution, is the branch and bound method. However, the time for solving the traveling salesman problem using the branch and bound method can vary from problem to problem even if their dimensions are the same.

In practice, when solving traveling salesman problems, cases are often found when an arc with a minimum weight is not included in the desired contour. Moreover, there are cases when none of the arcs with the smallest length in each row (column) of the matrix A, belongs to the optimal Hamiltonian contour. If this happens, the process of solving the problem by the branch and bound method is significantly delayed.

REFERENCES

1. *Sadigov A.B.* (2022). Assignment (selection) problem in operational and organizational control systems. The Reports of National Academy of Sciences of Azerbaijan, Vol. LXXVIII, (3-4), 3-8.
https://www.researchgate.net/publication/367523881_Zadaca_o_naznaceniah_v_sistemah_operativno-organizacionnogo_upravlenia

2. *Burda A.G., Burda G.P.* (2014). Operations research in the agro-industrial economy. Krasnodar. 566 p. (in Russian).
https://kubsau.ru/upload/iblock/97d/97de30615a3f14b5cbb505829bd94522.pdf

3. *Pintea C.M., Pop P.C., Chira C.* (2017). The generalized traveling salesman problem solved with ant algorithms. Complex Adaptive Systems Modeling, 5(8).
https://casmodeling.springeropen.com/articles/10.1186/s40294-017-0048-9#citeas

4. *Abdulkarim H.A., Alshammari I.F.* (2015). Comparison of algorithms for solving traveling salesman problem. International Journal of Engineering and Advanced Technology (IJEAT), 4(6), 76-79.
https://www.ijeat.org/wp-content/uploads/papers/v4i6/F4173084615.pdf

5. *Bandaliyev R.A., Guliyev V.S., Mamedov I.G., Sadigov A.B.* (2016). The optimal control problem in the processes described by the Goursat problem for a hyperbolic equation in variable exponent Sobolev spaces with dominating mixed derivatives. Journal of Computational and Applied Mathematics, (305), 11-17.
DOI: 10.1016/j.cam.2016.03.024

6.*Vyalitsin A.A.* (1991). On the enumeration of Hamiltonian cycles. Discrete Mathematics, 3(3), 46-49 (in Russian)
https://www.mathnet.ru/links/499f4b934c1f62dfdaae30fbaab96c54/dm803.pdf

7. *Sadigov A.B.* (2023). On the application of modified dynamic programming in operational and organizational control. Informatics and Control Problems, 43(1), 41-50.
https://icp.az/2023/1-06.php
DOI: https://doi.org/10.54381/icp.2023.1.06

8. *Kantsedal S.A., Kostikova M.V.* (2014). Dynamic programming for travelling salesman problem. Management Information System and Devices, (166), 15-20 (in Russian)
https://cyberleninka.ru/article/n/dinamicheskoe-programmirovanie-dlya-zadachi-kommivoyazhera

9. *Mustafayev I.I., Sadigov A.B.*, (2022). Modeling of regimes with sudden change. Reliability: Theory and Applications. 2022, Vol. 17, № 4(70), 111-117.
https://www.gnedenko.net/Journal/2022/SI_042022/RTA_SI_4_2022-10_111-117.pdf

10. *Boroznov V.O.* (2009). Study of the traveling salesman problem. Computer software and computer technology. Series “Management, Computer Engineering and Informatics”, (2), 147-151. (in Russian).
https://cyberleninka.ru/article/n/issledovanie-resheniya-zadachi-kommivoyazhera

11. *Silberholz J., Golden B.* (2007). The generalized traveling salesman problem: a new genetic algorithm approach. In book: Extending the Horizons: Advances in Computing, Optimization, and Decision Technologies. Operations Research/Computer Science Interfaces Series, (37), 165-181.
https://doi.org/10.1007/978-0-387-48793-9_11

12. *Sadigov A.B.* (2017). Models and technologies for

solving control problems in emergency situations. Baku, «Elm». 372 p. (in Russian). https://www.researchgate.net/publication/315893563_modelı_ı_tehnologıı_resenıa_zadac_upravlenıa_v_crezvycajnyh_sıtuacıah
https://isi.az/uploads/elmieserler/doc/aminaga_sadigov_20171657882973.pdf
13. *Karpov D.V.* (2018). Graph theory. Electronic resource. 563 p. (in Russian). https://logic.pdmi.ras.ru/~dvk/graphs_dk.pdf

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OPERATİV VƏ TƏŞKİLATİ İDARƏETMƏDƏ OPTİMAL MARŞRUTUN SEÇİLMƏSİ MƏSƏLƏSİ

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Optimal marşrutun seçilməsi məsələsinin qoyuluşunun üç ekvivalent forması nəzərdən keçirilir. Problemin həlli üçün dinamik proqramlaşdırma, budaq və sərhəd metodu, həmçinin evristik metodlardan istifadə edilmişdir. Budaq və sərhəd metodunun səmərəliliyini artırmaq üçün Hamilton konturunun bir sıra xüsusiyyətlərindən istifadə olunmuşdur.

Açar sözlər: *operativ idarəetmə, optimal marşrut seçilməsi məsələsi, dinamik proqramlaşdırma, budaq və sərhəd metodu, Hamilton konturu*

ЗАДАЧА ВЫБОРА ОПТИМАЛЬНОГО МАРШРУТА В ОПЕРАТИВНО-ОРГАНИЗАЦИОННОМ УПРАВЛЕНИИ

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Рассмотрены три эквивалентные формы постановки задачи выбора оптимального маршрута. Для решения задачи использовались метод динамического программирования, метод ветвей и границ, а также эвристические методы. Для повышения эффективности метода ветвей и границ использован ряд свойств контура Гамильтона.

Ключевые слова: *оперативное управление, задача выбора оптимального маршрута, динамическое программирование, метод ветвей и границ, гамильтонов контур*