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STOCHASTIC MODELS OF MOVING PARTICLES AND THEIR APPLICATION

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Stochastic models of moving particles, describing the various practical systems are constructed and investigated. Random errors, appearing in these systems (in transportation system changing of a speed, in communication systems delay of communication) transfer it into stochastic structure. In the paper, possible states appearing in such systems are described, an efficiency index for comparison these states is introduced. An optimal state has found and in comparison with determenistic models these state has quite different structure.

Keywords: *stochastic models, random delay function, regime of motion, stable regime, saturated and mixed regimes, optimal regime*

Introduction

Mathematical models of moving particles are used in the different applications (transportation, communication systems, network of computers and others). In [1-6] the various deterministic models of moving particles are introduced. These models have determenistic structure and a motion of the particles depends on a distance between them.

Although these models have determenistic structure it was succeded to observe some unexpected events, for instance, an appearance of a traffic jam [7,8]. However, the models, which can more adequatly describe a behavior of practical systems, have stochastic structure. In such models in random instants some changes are occured, which transfer the system from determenistic into stochastic. For instance, in transportation systems in some random instants transport unit can change a speed, which leads to changing of a state of system motion. Hence, system transfers from determenistic into stochastic systems. Such models attract attention of the various researchers and scholars. Unfortunately, these processes have complicated structure and usage of well known methods, for instance theory of Markov processes [9] is faced with difficulties and more over sometimes their structure is not Markovian. Hence, it is necessary to develope the new methods and approaches for investigation of such systems. Some approaches for investigation of the systems with cyclic services, which can be used for these models are given in [10,11].

Construction of stochastic models

Consider on a curve of the length one unit k particles ($k > 1$), which are moving in one direction. For construction of mathematical models according to [1,10] the following notations are introduced:

$\xi_{i,t}$ – is coordinate of i-th particle at the instant t; each particle (depending on a distance to the next) moves either with the speed V_1 or V_2;

$V_{i,t}$ – is the speed of i-th particles at the instant t;

$\rho_{i,t} = \xi_{i+1,t} - \xi_{i,t}$ – distance between i-th and ($i + l$)-th particles at the instant t, for which we have

$$\rho_{i,t} = \xi_{i+1,t} - \xi_{i,t}, \text{ if } \xi_{i+1,t} > \xi_{i,t}$$
$$\rho_{i,t} = n - (\xi_{i,t} - \xi_{i+1,t}) \quad \text{if} \quad \xi_{i+1,t} < \xi_{i,t},$$
$i = 1, 2, ..., k-1$;
$$\rho_{k,t} = \xi_{1,t} - \xi_{k,t} \text{ if } \xi_{1,t} > \xi_{k,t}$$
$$\rho_{k,t} = n - (\xi_{k,t} - \xi_{1,t}) \text{ if } \xi_{1,t} < \xi_{k,t}$$

It is supposed also that there exist minimal admittable distance between particles, which does not allow for particles to overtake each other. For defining a speed of particle we introduce the quantities Q_1 and Q_2 and assume that ($0<Q_1<Q_2<1$). If $\rho_{i,t}=Q_1$ and $\rho_{i,t}$ is increasing (it can be happened if i-th particle has the speed V_1 and $(i+1)$-th particle has the speed V_2), then if there exists some t^*, for which $\rho_{i,t^*}=Q_2$, then at the instant t^*, i-th particle changes a speed from V_1 to V_2. If $\rho_{i,t}=Q_2$ and $\rho_{i,t}$ is decreasing (it can be occured if i-th particle has the speed V_2 and $(i+1)$-th particle has the speed V_1), if there exists some t^{**}, for which $\rho_{i,t^{**}}=Q_1$ then at the instant t^{**}, i-th particle changes speed from V_2 to V_1. Thus, particles change speeds at the end points of the interval $[Q_1,Q_2]$. Similarly, as it was introduced in [12] we remind the following definitions.

Definition 1. If for any t all distances between $(k-l)$ particles equal Q_1, then this state is called zero state.

Definition 2. If for some t we have $V_{i,t}=V_1, i=1,2,...,k$; then this state is called stable regime 1 and is denoted (*sr1*).

Definition 3. If for some t we have $V_{i,t}=V_2, i=1,2,...,k$; then this state is called stable regime 2 and is denoted (*sr2*).

Definition 4. If for some t we have $V_{i,t}=V_1, i=1,2,...,k_1$ and

$V_{j,t}=V_1,\ j=k_1+1,k_1+2,...,k;(k_2=k-k_1)$

then this state is called mixed statble regime and is denoted (*msr*).

Definition 5. If for some t we have $\rho_{i,t}=Q_1+\varepsilon_i$, $V_{i,t}=V_2, i=1,2,...,k$ and $Q_1<\varepsilon_1+\varepsilon_2+...+\varepsilon_k<Q_2$ this state is called saturated stable regime and is denoted (*ssr*).

Definition 6. If at the random instants $t_1,t_2,...,t_n$ some particles change speed from V_2 to V_1, then we are saying that delays are occured in the system.

Definition 7. If after occuring of delays, a system can come back to the state, where it was before delays, then such delays are called *nonessential* and regime of motion is called *invariant* regime (*ir*). It is clear that delays can be introduced only for systems, where at least one particle has the speed V_2, hence for (*ssr1*) it is not necessary to introduce it.

Example 1. Assume, that system operates in (*tsr2*) rejime, $\rho_{1,t}=2Q_2$ and at the instant t it is occured one delay with long a $(Q_2-Q_1)/(V_2-V_1)$, i.e. a speed of the second particle is changed to V_1 and during the time t^* $(t^*<(Q_2-Q_1)/(V_2-V_1))$ the speed of second particle equals V_1.

According to construction of the model we have

$$\rho_{1,t+t^*}=2Q_2-(V_2-V_1)(Q_2-Q_1)/(V_2-V)=$$
$$=2Q_2-Q_2+Q_1=Q_2+Q_1>Q_2$$
$$\rho_{2,t+t^*}=Q_2+Q_2-Q_1>Q_2,$$
$$\rho_{i,t+t^*}=Q_2, i=3,4,...,k.$$

Thus, system still operates in the (*sr2*). It means that this delay was nonessential and (*tsr2*) is invariant state relatively to this delay.

Example 2. Consider two regimes of motion (*sr2*) and (*msr*) and for both regimes we will introduce the delays. We also assume that

$$\max(\varepsilon_i)<t^*<(Q_2-Q_1)/(V_2-V_1),$$

i.e. $\varepsilon_i>0$ (takes small values) as it was introduced for saturated regime (*ssr*) in [12]. If in a saturated regime at least one delay with long t^* is occured, then system changes its regime of motion from (*ssr*) to (*sr1*). This delay is essential for saturated regime and is nonessential for (*sr2*), because for we have

$$\rho_{1,t}>Q_1, \rho_{2,t}>Q_1,...,\rho_{k,t}>Q_1,$$

which means that all particles will have speed V_2.

Suppose, that in the system only one delay is occured, i.e. in some random instant the speed of one particle is changed from V_2 to V_1. Then the following theorem is true.

Theorem 1. If in the system only one delay is occured, then

The regimes (*sr2*) and (*msr*) are invariant

If

$$Q_1 << \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_k < Q_2 \ , \qquad (1)$$

then (*ssr*) passes to (*sr1*) and stay there all time

If

$$\varepsilon_1 + \varepsilon_2 + ... + \varepsilon_k > Q_2 \, , \qquad (2)$$

then (*ssr*) regime passes to (*msr*) and stay there all time.

Proof. If only one delay is occured in the system, then after delay system becomes determenistic.

According to [12], for determenistic systems, for operating in (*sr2*) it is necessary and sufficient to have the following conditions

$$(k-1)Q_2 + Q_1 < 1. \qquad (3)$$

If a system is operating in the (*sr2*) regime, then the condition (3) is held and after delay the system is still determenistic. Then after some time it will come again to the regime (*sr2*). According to [12], for determenistic systems, for operating in (*msr*) it necessary and sufficient to have the following condition

$$(k_1 - 1)Q_1 + (k_2 + 1)Q_2 < 1. \qquad (4)$$

If system is operating in the (*msr*) regime, then system is still determenistik and the condition (4) is held, then after some time it will come again to the regime (*msr*).

If

$$Q_1 < \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_k < Q_2,$$

then after finishing a delay, system again becomes determenistic and passes to the regime (*ssr1*), because all conditions for operating in this regime are held.

If

$$\varepsilon_1 + \varepsilon_2 + ... + \varepsilon_k < Q_2,$$

then after occuring of delay, system passes to zero state and from this state after some time it comes to (*msr*) regime, because according to the condition (4) a distance between at least two particles equals or greater than Q_2 and hence at least one particle has the speed V_2, i.e. we have

$$\rho_{i,t^*} = Q_2 + t^*(V_2 - V_1) > Q_2, V_{i,t^*} = V_2,$$
$$\rho_{i-1,t^*} = Q_2 + t^*[(Q_2 - Q_1)/(V_2 - V_1)](V_2 - $$
$$- V_1) > Q_1, V_{i-1,t^*} = V_2$$

Thus, $V_{i-1,t^*} = V_2$ and system is operating in the regime (*sr2*).

If $t^* \geq (Q_2 - Q_1)/(V_2 - V_1)$, then after time t^* we have

$$\rho_{i,t^*} = Q_2 + t^*(V_2 - V_1) > Q_2, V_{i,t^*} = V_2,$$
$$\rho_{i-1,t^*} = Q_2 + t^*[(Q_2 - Q_1)/(V_2 - V_1)](V_2 - $$
$$- V_1) > Q_1, V_{i-1,t^*} = V_1.$$

If a long of delay equals or greater than $(k-1)(Q_2 - Q_1)/(V_2 - V_1)$, tin a system hence system passes to zero state and from zero state passes into (*sr2*). Cases (b) and (c) have a similar proof. (ii)–part. Assume that system is operating in (*ssr*) and i-th particle changes the speed from V_2 to V_1. This (i-th) particle never change speed to V_2 because for any t we have $\rho_{i,t} < Q_2$ and after time $\varepsilon_i/(V_2 - V_1)$ $(i-1)$-th particle change speed to V_1 and this process will be continued until an instant when all particles will have the speed V_1. Thus, system will come to the regime (*sr1*), because after the time $(\varepsilon_1+\varepsilon_2+...+\varepsilon_i)/(V_2-V_1)$ the sistem is operating in the regime (*sr2*), i.e. $V_{i-1,t}=V_2$ and system is operating in the regime (*sr2*). Thus, if in (*ssr*) only once some particle changes speed from V_2 to V_1 the it will keep this speed all time. It is necessary to note, that if in (*ssr*) will occued any delay then system passes to (*sr1*) and never back again. It means that for some cases saturated regime is not recommended for practice. Hence, if it is expecting delays in the system, then (*ssr*) is not optimal and preferable to have (*sr2*). Let us introduce additional conditions. At the instant t some delays can be happened in the system, i.e. some particle having speed V_2 will change it to V_1.

Theorem 2. If the condition (i) is held then the following inequality is true

$$hgm_{ssr2}\le hgm_{M(k_1ck_{s2})}\le hgm_{msr}=hgm_{ssr1}$$

Proof. Assume that in (*ssr*) at the instant t^* i-th particle will change speed from V_2 to V_1, i.e. $\rho_{1,t^*}=Q_1+\varepsilon_1, V_{1,t}=V_2$,

$$\rho_{2,t^*}=Q_1+\varepsilon_2, V_{2,t}=V_2,...,$$
$$\rho_{i-1,t^*}=Q_1+\varepsilon_{i-1}, V_{i-1,t^*}=V_2,$$
$$\rho_{i,t^*}=Q_1+\varepsilon_i, V_{i,t^*}=V_1, \varepsilon_i>0$$
$$\rho_{i+1,t}=Q_1+\varepsilon_{i+1}, V_{1,t}=V_2,...,$$
$$\rho_{k,t^*}=Q_1+\varepsilon_k, V_{k,t^*}=V_2$$ then after time $\varepsilon_i/(V_2-V_1)$, i.e. at the instant $t^{**}=t^*+\varepsilon_{i-1}/(V_2-V_1)$ we have $\rho_{i-1,t^{**}}=Q_1$ and hence $V_{i-1}=V_1$. Step by step distances between $(k-1)$ particles will be equal Q_1, but only one distance according to the condition (1) will be equal or greater than Q_1, but less than Q_2, hence the system will operating in the (*ssr1*). If in (*tsr2*) regime at the instant t^* delay is occured, i.e. i-th particle changes speed from V_2 to V_1 then we have

$$\rho_{1,t^*}=Q_2, V_{1,t^*}=V_2; \rho_{2,t^*}=Q_2, V_{2,t^*}=V_2;...,$$
$$\rho_{i-1,t^*}=Q_2, V_{i-1,t^*}=V_2; \rho_{i,t^*}=Q_2, V_{i,t^*}=V_1,$$
$$\rho_{i+1,t^*}=Q_2,\ V_{i+1,t^*}=V_2;...;\rho_{k,t^*}=Q_2, V_{k,t^*}=V_2.$$

Long delay forces system to pass into the zero state and for any t^* we have

$$\rho_{1,t^*}=Q_1, V_{1,t^*}=V_1; \rho_{2,t^*}=Q_1, V_{2,t^*}=V_1;...,$$
$$\rho_{s-1,t^*}=Q_1, V_{s-1,t^*}=V_1; \rho_{s,t^*}\ge Q_2, V_{s,t^*}=V_2$$

As system is operating in the regime (*ssr2*) then the following condition $(1-Q_2-Q_1)/Q_1\le k<1/Q_1$ is true. Let us use Theorem 2. As all conditions are held it follows that system is operating in (*sr2*).

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HƏRƏKƏTDƏ OLAN HİSSƏCİKLƏRİN STOXASTİK MODELLƏRİ VƏ ONLARIN TƏTBİQİ

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Hərəkətdə olan hissəciklərin stoxastik modelləri qurulur və araşdırılır. Belə modellərdə yaranan təsadüfi maneələr (nəqliyyat məsələlərində sürətin azalması, rabitə sistemlərində əlaqələrin gecikdirilməsi və s.) sistemlərə stoxastik xarakter gətirir və onların araşdırılması üçün yeni yanaşmaların və üsulların yaradılmasını tələb edir. Məqalədə təklif olunan yeni yanaşmalar qurulan modellərdə yarana bilən vəziyyətləri tapmağa imkan verir və sistemləri müqayisə etmək üçün effektivlik göstəricisi (təsadüfi seçilmiş nöqtədə hissəciyin orta gözləmə müddəti) daxil edilir. Optimal hərəkət rejimləri tapılır və göstərilir ki, determinik sistemlərdən fərqli olaraq bu rejimlər tam fərqli xarakter daşıyır.

Açar sözlər: *stoxastik modellər, təsadüfi gecikdirmə funksiyası, hərəkət rejimləri, doymuş və qarışıq rejimlər, optimal rejim*

СТОХАСТИЧЕСКИЕ МОДЕЛИ ДВИЖУЩИХСЯ ЧАСТИЦ И ИХ ПРИМЕНЕНИЕ

Н.А.Абдуллаева

Построены и исследованы стохастические модели движущихся частиц. Возникающие в таких моделях случайные помехи (на транспорте резкое изменение скорости, в системах связи задержка связи и др.) придают стохастический характер этим системам, и требуются разработки новых методов и подходов для их исследования. Предложен новый подход для исследования таких систем, позволяющий описать класс возникающих режимов движения, и для сравнения таких режимов введен показатель эффективности (среднее время ожидания частицы в случайно выбранной точке. Найдены оптимальные режимы движения и показано, что в отличие от детерминированных систем они имеют принципиально разную структуру.

Ключевые слова: *процесс восстановления, функция восстановления, процесс восстановления-вознаграждения, математическое ожидание, условие Крамера*