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APPROXIMATE SOLUTIONS OF THE INTERVAL PROBLEM OF MIXED INTEGER PROGRAMMING

K.Sh.Mammadov[1], N.O.Mammadli[2]
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In this article the problem of mixed-integer programming with interval initial data is considered. The concepts of feasible, optimistic, pessimistic, suboptimistic and sub-pessimistic solutions were introduced in the considered problem. Based on the economic interpretation of the problem, the methods were proposed finding suboptimistic and sub-pessimistic solutions.

Keywords: *an interval problem of mixed-integer programming, feasible, optimistic, pessimistic, suboptimistic and sub-pessimistic solutions*

Introduction

The following interval problem of mixed-integer programming is considered:

$$\sum_{j=1}^{n}\left[\underline{c}_j,\bar{c}_j\right]x_j+\sum_{j=n+1}^{N}\left[\underline{c}_j,\bar{c}_j\right]x_j\to\max \qquad (1)$$

$$\sum_{j=1}^{n}\left[\underline{a}_{ij},\bar{a}_{ij}\right]x_j+\sum_{j=n+1}^{N}\left[\underline{a}_{ij},\bar{a}_{ij}\right]x_j\le\left[\underline{b}_i,\bar{b}_i\right],(i=\overline{1,m}), \qquad (2)$$

$$0\le x_j\le d_j,(j=\overline{1,N}), \qquad (3)$$

$$x_j \text{-integer}, \ (j=\overline{1,n}),(n\le N). \qquad (4)$$

Here it is assumed that the coefficients of the problem (1) - (4) are integers and satisfy the following conditions $0<\underline{c}_j\le\bar{c}_j$, $0\le\underline{a}_{ij}\le\bar{a}_{ij}$, $0<\underline{b}_i\le\bar{b}_i$, $d_j,(i=\overline{1,m};\ j=\overline{1,N})$ .In addition, the following natural conditions for the coefficients of the problem (1)-(4) must be fulfilled:

$$\sum_{j=1}^{N}\underline{a}_{ij}d_j>\bar{b}_i,(i=\overline{1,m}).$$

And if for all $i,(i=\overline{1,m})$ this condition is not fulfilled, then the solution $X=(d_1,d_2,...,d_N)$ will be the best solution, i.e. optimal solution of the problem (1)-(4). It should be noted that if for some number i_* the inequality $\sum_{j=1}^{N}\bar{a}_{i_*j}d_j\le\bar{b}_{i*}$ holds, then this is not a restriction for system (2) and it can be omitted. Therefore, here we assume that for the problem (1)-(4) the above conditions are fulfilled.

It should be noted that the problem (1) - (4) is called the problem of mixed-integer programming with interval data or the interval problem of mixed-integer programming.

It is appropriate to note that problem (1) - (4) is a generalization of: 1) Boolean programming problem, 2) integer programming, 3) interval Boolean programming problem, 4) interval integer programming problem, 5) interval mixed-Boolean programming problem, 6) linear programming problem, 7) interval linear programming problem.

Because 1) in cases where the ends of the intervals coincide, $n=N$ and $d_j=1,(j=\overline{1,N})$ the Boolean programming problem is obtained, 2) if $n=N$ and the ends of the intervals coincide, the integer programming problem is obtained, 3) if $d_j=1,(j=\overline{1,N})$ and $n=N$ interval Boolean programming problem is obtained, 4) if $n=N$ interval integer programming problem is obtained, 5) if $d_j=1,(j=\overline{1,N})$ and $n<N$ interval mixed-Boolean programming problem is obtained, 6) if the ends of the intervals coincide and $n=0$ linear programming problem is obtained, 7) if $n=0$ interval linear programming problem is obtained.

We note that the considered problem (1) - (4) belongs to the class NP-complete, since all of the above particular cases of this problem, besides the linear programming problems, are NP-complete. In other words, difficult-to-solve [1; 2]. Different classes of interval integer programming problems were investigated and specific methods were developed in [3-9]. The interval problems of linear (non-integer) programming were investigated in [10,11,12 and others].

It should be noted, as far as we know, the problem of mixed-integer programming with interval data has not yet been investigated. This may be due to the complexity of developing methods of their exact solution.

In this article, new concepts of feasible, optimistic, pessimistic, suboptimistic and subpessimistic solutions for problem (1) - (4) were introduced and an algorithm for its approximate solution was developed. And in doing so, we used the principles of interval calculi introduced in [13].

Problem statement

First, let us present some applications of the considered model
(1)-(4). Suppose that an enterprise produces N types of goods. Let them be n $(n \le N)$ types of piece goods, and for $N-n$ non-piece goods.

Let for the production of these goods, the m types of resources were allocated included in the interval $[\underline{b}_i, \bar{b}_i], (i=\overline{1,m})$. In addition, let us assume that for the production of the unit j-th goods, the expenses included in the interval $[\underline{a}_{ij}, \bar{a}_{ij}], (i=\overline{1,m};\ j=\overline{1,N})$ are required. We assume that the profit from the sale of each j-th item of the goods enters the interval $[\underline{c}_j, \bar{c}_j], (j=\overline{1,N})$. Let it be required to find such quantities of planned types of piece and non-rodent goods, for which the total expenses did not exceed the allocated resources $[\underline{b}_i, \bar{b}_i], (i=\overline{1,m})$, respectively. At the same time, the total profit was maximal. If we take unknowns x_j $(j=\overline{1,N})$, where $0 \le x_j \le d_j, (j=\overline{1,N})$, and x_j - integer, $(j=\overline{1,n};\ n \le N)$, then we get model (1) - (4).

It should be noted that in the areas of production, where goods are manufactured, some of which must be piece, the considered model (1) - (4) necessarily takes place.

Theoretical justification of the method

Before presenting the method for solving problem (1) - (4), we introduce some definitions. These definitions are more generalized than the definitions introduced in [14] for the interval problem of mixed-Boolean programming.

Definition 1. A certain vector $X=(x_1,x_2,...,x_N)$ is called a feasible solution of the problem (1)-(4) that satisfies the system (2)-(4) for $\forall a_{ij} \in [\underline{a}_{ij}, \bar{a}_{ij}]$ and $\forall b_i \in [\underline{b}_i, \bar{b}_i]$, $(i=\overline{1,m};\ j=\overline{1,N})$.

Obviously, for the solution of problem (1) - (4), it is necessary to ensure the non-exceedance condition of sum of the several different intervals from a given interval, while achieving the maximum of the corresponding intervals.

Definition 2. A feasible solution $X^{op}=(x_1^{op},x_2^{op},...,x_N^{op})$ is called an optimistic solution of the problem (1)-(4) in case for $\forall b_i \in [\underline{b}_i, \bar{b}_i]$, $(i=\overline{1,m})$ the inequalities $\sum_{j=1}^{N} \underline{a}_{ij} x_j^{op} \le b_i$, $(i=\overline{1,m})$ are fulfilled and in this, the value of a function $f^{op}=\sum_{j=1}^{N} \bar{c}_j x_j^{op}$ will be maximal.

Definition 3. A feasible solution $X^{p}=(x_1^{p},x_2^{p},...,x_N^{p})$ is called an optimistic solution of the problem (1)-(4) in case for $\forall b_i \in [\underline{b}_i, \bar{b}_i]$, $(i=\overline{1,m})$ the inequalities $\sum_{j=1}^{N} \bar{a}_{ij} x_j^{p} \le b_i$, $(i=\overline{1,m})$ are fulfilled and in this, the value of a function $f^{p}=\sum_{j=1}^{N} \underline{c}_j x_j^{p}$ will be maximum.

From the last two definitions it turns out that the problem of finding the optimistic and pessimistic solutions of the interval problem (1) - (4) also belongs to the class NP-complete, since the particular cases of this problem are NP-complete. Therefore, we developed methods for

the approximate solution of the interval problem of mixed integer programming and called it **suboptimistic** and **sub-pessimistic solutions.**

In order to develop method for constructing suboptimistic and sub-pessimistic solutions, we will use the economic interpretation of problem (1) - (4) introduced in problem statement.

Suppose that some j-th product is produced. Then the necessary resource from the allocated total resource $[\underline{b}_i, \bar{b}_i], (i=\overline{1,m})$ will be included in the interval $[\underline{a}_{ij}, \bar{a}_{ij}], (i=\overline{1,m}; j=\overline{1,N})$. At the same time, the profit of this j-th product should be included in the interval $[\underline{c}_j, \bar{c}_j], (j=\overline{1,N})$. Obviously, the profit of the objective function for each unit of expenditure for the j-th product $(j=\overline{1,N})$ will be at least

$$\min_i \frac{[\underline{c}_j, \bar{c}_j]}{[\underline{a}_{ij}, \bar{a}_{ij}]} = \frac{[\underline{c}_j, \bar{c}_j]}{\max_i [\underline{a}_{ij}, \bar{a}_{ij}]}, \ (j=\overline{1,N}). \quad (5)$$

From this relation, it follows that it is necessary to produce such a product with a number j_* for which (5) will be maximal:

$$\max_j \frac{[\underline{c}_j, \bar{c}_j]}{\max_i [\underline{a}_{ij}, \bar{a}_{ij}]} = \frac{[\underline{c}_{j*}, \bar{c}_{j*}]}{\max_i [\underline{a}_{ij*}, \bar{a}_{ij*}]}. \quad (6)$$

Relation (6) shows that in order to construct suboptimistic and sub-pessimistic solutions, the number j_* must be determined from the following relations, respectively:

$$\max_j \frac{\bar{c}_j}{\max_i \underline{a}_{ij}} = \frac{\bar{c}_{j*}}{\max_i \underline{a}_{ij*}}, \quad (7)$$

$$\max_j \frac{\underline{c}_j}{\max_i \bar{a}_{ij}} = \frac{\underline{c}_{j*}}{\max_i \bar{a}_{ij*}}. \quad (8)$$

Obviously, when constructing suboptimistic and sub-pessimistic solutions on the basis of criteria (7) and (8), it is necessary to take into account the circumstance in which interval the number j_* is included in i.e. $j_* \in I = [1,...,n]$ or $j_* \in R = [n+1, n+2,..., N]$.

In this article, taking these circumstances into account, two methods for constructing suboptimistic and sub-pessimistic solutions of problem (1) - (4) were developed.

In the first method, if $j_* \in R$ and it is impossible to assign d_{j*} to the unknown x_{j*}, then for x_{j*} we take the best possible value, and the remaining unknowns will take the value of zero.

And in the second method, if $j_* \in R$ and to the unknown x_{j*} it is impossible to assign d_{j*} then we fix all found values until now but for the remaining numbers $j \in I$ we accept $x_j := 0$ and for x_j, where $j \in R$ we build the linear programming problem of smaller dimension. Solving the obtained linear programming problem of smaller dimension, we join the obtained solutions to the previously fixed ones. Note that in both methods, at the beginning of the solution process, we take $X^{so} == (0,0,...,0)$ and $X^{sp} == (0,0,...,0)$.

The conducted computational experiments once again confirmed the high efficiency of the proposed methods.

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[1] Baku State University
[2]Institute of Control Systems of ANAS

İNTERVALLI QİSMƏN TAMƏDƏDLİ PROQRAMLAŞDIRMA MƏSƏLƏSİNİN TƏQRİBİ HƏLLİ

K.Ş.Məmmədov, N.O.Məmmədli

Məqalədə intervallı qismən tamədədli proqramlaşdırma məsələsinə baxılmışdır. Bu məsələ üçün mümkün olan optimist, pessimist, suboptimist və subpessimist həll anlayışları verilmişdir.

Baxılan məsələnin iqtisadi interpretasiyasına əsaslanaraq, onun təqribi suboptimist və subpessimist həllinin tapılması üsulu təklif olunmuşdur.

Açar sözlər: *intervallı qismən tamədədli proqramlaşdırma məsələsi, mümkün həll, optimist, pessimist, suboptimist və subpessimist həllər*

ПРИБЛИЖЕННЫЕ РЕШЕНИЯ ИНТЕРВАЛЬНОЙ ЗАДАЧИ ЧАСТИЧНО-ЦЕЛОЧИСЛЕННОГО ПРОГРАММИРОВАНИЯ

К.Ш.Мамедов, Н.О.Мамедли

В работе рассматривается задача частично-целочисленного программирования с интервальными данными. Введены понятия допустимого, оптимистического, пессимистического, субоптимистического и субпессимистического решений в рассмотренной задаче. Исходя из экономической интерпретации задачи, предложены методы нахождения субоптимистического и субпессимистического решений.

Ключевые слова: *интервальная задача частично-целочисленного программирования, допустимое, оптимистическое, пессимистическое, субоптимистическое и субпессимистическое решения*