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NEW INVERSE PROBLEM TO DETERMINE THE ORDER FRACTIONAL DERIVATIVES OF THE OSCILLATION SYSTEM

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For the first time, the formulation of new inverse problems is given to determine the fractional derivatives order in the subordinate terms of the oscillation process. A method is proposed for determining the above parameter based on the statistical data of the final values of the corresponding coordinate and the least squares method. The result is illustrated by a numerical example.

Keywords: *oscillation process, inverse problem, fractional derivative, statistical data, least-squares method*

Introduction

As is known, the motion of oscillation systems inside a fluid is described by the following second-order differential equation with fractional derivatives [1,2]

$$y''(x)+aD^{\alpha}_{x_0+}y(x)+by(x)=f(x),\ (x\ge x_0>0) \quad (1)$$

and initial condition

$$y(x_0)=y_0,\ y'(x_0)=y_1,\ \alpha\in(1,2), \quad (2)$$

where $a=\dfrac{2s\sqrt{\mu\rho}}{m}$, $b=\dfrac{k}{m}$, in the Fig. 1 m is mass, s - hard plate area, ρ - density, μ - Newtonian fluid viscosity, k - spring constant, $f(t)$ - external force.

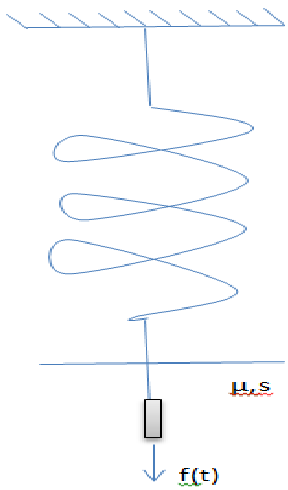

Fig.1

Note that this problem describes the motion of oscillation systems when the mass is inside the Newtonian fluid [1] (in the classical version, i.e. there is no liquid $\alpha=1$). To such an event, when extracting oil, the movement of the pumping unit, where the plunger moves inside Newtonian fluid (oil) [1,3,4]. Usually for such problems from (1) a,b,f are considered known and constant, and as for the order of the fractional derivative α, it depends on these data (a,b,f includes the mass of oscillation systems m, the viscosity of the fluid, the coefficient of elasticity of the spring, the density of the fluid, etc.). Therefore, it is convenient to determine the parameter from the statistical data of motion, and it depends on these data in an implicit form. Here, instead of this data, at different initial values $y^i(x)$ ($i=\overline{1,k}$) we can take the finite values $y^i(x,\alpha_k)$, where x is the finite value of the argument, α_k is the parameter value α, which is depends on a,b,f.

In this work based on statistics $y^i(x_0)$, $y^i(x,\alpha_i)$ a quadratic functional is constructed on the basis of the given statistical data and the solution of the problem (1), (2). To construct a solution with unknown α the definition of fractional order derivatives of Riemann-Liouville is used, where the solution of the problem is reduced to solving the II type Voltaire integral equation [5]. Using the method of successive statements $y(x,\alpha_i)$ is constructed approximately and a nonlinear algebraic equation is given to determine of α. The results are illustrated for specific values *a,b,f* and α using the method of separation of the segment of definitions $[x_0,x]$ in half, where α is found with accuracy 10^{-5}.

Reduction of the problem (1), (2) to the II type Volterra integral equation. Using the Riemann-Liouville definition [6]

$$D_{x_0}^{\alpha} y(x) = \frac{d^2}{dx^2}\int_{x_0}^{x}\frac{(x-t)^{1-\alpha}}{(1-\alpha)!}y(t)dt, \alpha \in (1,2),\ x \ge x_0 > 0 , \quad (3)$$

equation (1) can be reduced to

$$y'' + a\frac{d^2}{dx^2}\int_{x_0}^{x}\frac{(x-t)^{1-\alpha}}{(1-\alpha)!}y(t)dt + by(x) = \mathrm{f}(x), \quad (4)$$

where after some simple transformations, (4) is reduced to the form

$$y(x) - y_1(x-x_0) + a\int_{x_0}^{x}\frac{(x-t)^{1-\alpha}}{(1-\alpha)!}y(t)dt +$$

$$+b\int_{x_0}^{x}(x-t)y(t)dt = \int_{x_0}^{x}(x-t)f(t)dt. \quad (5)$$

Here, for simplicity of presentation, it is assumed that the origin of coordinates is placed so that $y(x_0)=0$, then equations (5) can be written in the form

$$y(x) + \int_{x_0}^{x} K_\alpha(x-t)y(t)dt = F(x), \quad (6)$$

where (6) is the II type Volterra integral equation

$$K_\alpha(x-t) = a\frac{(x-t)^{1-\alpha}}{(1-\alpha)!} + b(x-t), \quad (7)$$

$$F(x) = \int_{x_0}^{x}(x-t)f(t)dt + y_1(x-x_0). \quad (8)$$

As is known (6), the solution of equation (6) is analytically represented as a Neumann series (solution through a resolvent). For simplicity, we discretize the integral equation (6) in the following form

$$y_i + \sum_{k=0}^{i-1} K_\alpha(x_i - x_k)y_k(x_k)\cdot h = F_i, \qquad i \ge 1 , \quad (9)$$

where

$$K_\alpha(x_i - x_k) = a\frac{(x_i-x_k)^{1-\alpha}}{(1-\alpha)!} + b(x_i - x_k),\ k = \overline{0, i-1}, \quad (10)$$

$$F_i(x_i) = \sum_{k=0}^{i-1}(x_i - x_k)f(x_k)\cdot h + y_1(x_i - x_0) , \quad (11)$$

an interval $(x_0 \quad x)$ is divided in steps h.

Note that (9) is a numerical solution of problem (1), (2).

Construction of a quadratic functional on the basis of statistical data $y^i(x_0)$

With statistical data $y^i(x_0)$ and $y^i(x_n)$ $\left(x = x_n,\quad i = \overline{1}, \overline{k}\right)$ $_1$let compose a quadratic functional.

$$J = \sum_{i=1}^{k}\left(y^i(x_n,\alpha) - y^i(x_n)\right)^2 . \quad (12)$$

Note that, the form of $y^i(x_n,\alpha)$ is determined from (9) - (11) and the information about α is hidden in $y^i(x_n)$. From the convexity of the functional (12) we can easily find α through the minimization of the functional (12), i.e., the extremum is determined from an algebraic equation

$$\frac{\partial J}{\partial \alpha} = 0, \quad (13)$$

where (13) can be written as

$$\sum_{i=0}^{k}\left(y^i(x_n,\alpha) - y^i(x_n)\right)\cdot\frac{\partial y^i(x_n,\alpha)}{\partial\alpha} = 0 , \quad (14)$$

here

$$\frac{\partial y^i(x_n,\alpha)}{\partial\alpha} =$$

$$= -\sum_{k=0}^{n-1} a\frac{(x_n-x_k)^{1-\alpha}\ln(x_n-x_k)\Gamma(2-\alpha) - (x_n-x_k)^{1-\alpha}\Gamma'(2-\alpha)}{\Gamma^2(2-\alpha)}, \quad (15)$$

Solving equation (14) for α, we determine the fractional derivative α from problems (1) - (2).

Example. Consider the problems (1) - (2) with a = 1, b = 1, f = 8 on the interval (0 1), (1,1). Dividing it into 10 parts, we set the statistical data y_i^k in the following table.

Table

i \ m	0	1	2	3	4	5	6	7	8	9	10
1	0	-0.67	-1.37	-1.13	-1.00	-0.91	-0.84	-0.79	-0.74	-0.71	-0.67
2	0	-0.67	-2.45	-1.58	-1.15	-0.88	-0.68	-0.53	- 0.41	-0.31	-0.22
3	0	-0.67	-3.19	-1.62	-0.93	-0.51	-0.23	-0.02	0.13	0.26	0.37
4	0	-0.67	-2.86	-0.93	-0.16	0.25	0.53	0.72	0.87	0.98	1.08
5	0	-0.67	-0.34	0.81	1.22	1.44	1.57	1.66	1.72	1.77	1.81

After solving equations (14), (15) on the basis of statistical data from the table, we obtain $\alpha* = 1.853$ (see, Fig.2). Note that the solution of equation (14) was obtained using the procedure of dividing the interval (1,2) in half [7].

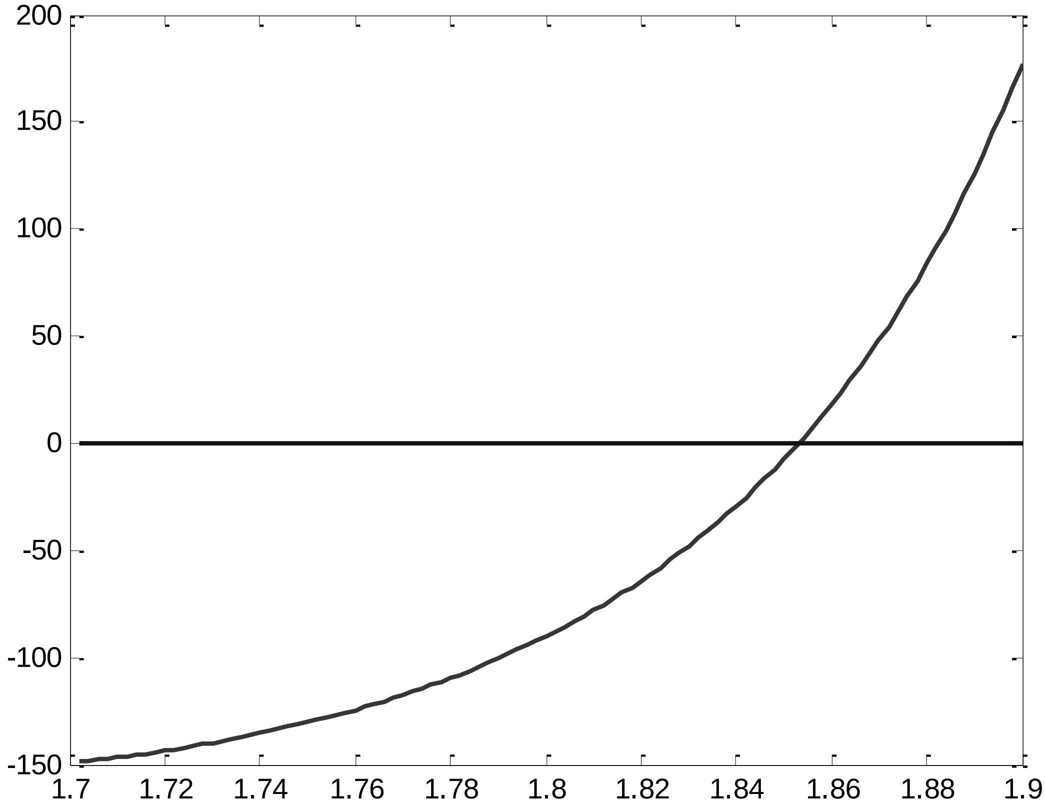

Fig. 2. Dependence $y(\alpha)$ on α

Conclusion

For the first time, on the basis of given statistical data, a new generation of inverse problems is mathematically considered to determine the order of fractional derivatives problems of oscillation processes. Note that the first generation of inverse problems is the Tikhonov – Lavrent'ev problem, in which, besides the basic unknown, the coefficients of the equation or boundary condition or their right-hand sides are defined, where the inverse problems of the theory of scattering are a special case of the above. The second generation of inverse problems are Stefan problems, where, in addition to the main unknown, either a part or the entire boundary is searched. Thus, the work outlined here relates to the third generation of the inverse problem. On the basis of these studies, various problems of the control problem can be considered, for example, the construction of program motions and control [8-12], optimal stabilization [12, 13], and others.

REFERENCES

1. *Bonilla B., Rivero M., Trujillo J.J.* On systems of linear fractional differential equations with constant coefficients. Appl. Math. Comput., 2007, v.187, pp.68-78

2. *Monje C. A., Chen Y. Q., Vinagre B. M., Xue D., Felue V.*, Fractional-Order Systems and Controls. Fundamentals and Applications.Springer, (2010) (London) 414 p.

3. *Aliev, F.A., Mutallimov, M.M.* Method of solving optimization problems in the operation of oil wells. LAP LAMBERT, 2012 (in Russian)

4. *Aliev F.A., Abbasov A.N., Mutallimov M.M.* Algorithm for the solution of the problem optimization of the energy expenses at the exploitation of chinks by subsurface-pump installations. Appl. And Comp. Math. 2004, 3(1), pp.2-9.

5. *Petrovsky I.G.* Lectures on the theory of integral equations. M .: Nauka, 1965. - 128 p.

6. *Samko S.G, Kilbas A.A., Marichev O.I.* Fractional integrals and derivatives: Theory and applications, Gordon and Breach Science publishers, Yverdon, Switzerland, 1993, 780 p.

7. *Himmelblau D.M.* Applied Nonlinear Programming, New-York, Craw-Hill Book Company, 1972, p.536

8. *Aliev F.A.* Methods to solve applied problems of optimization of dynamic system. Baku: Elm,1989.

9. *Aliev, F.A.* The problem of optimal control of a linear system with unseparated two-point boundary conditions. Differential Equations, 22 (2), 1986, p. 345-347 (in Russian).

10. *Aliev F.A.* Optimization problem with point-to-point boundary conditions. Proceedings of the USSR Academy of Sciences, ser. Tech. Cybernetics, 1985, p. 138 (in Russian).

11. *Maksudov F.G., Aliev F.A.* Optimization of the impulse systems with the nondivided boundary-value conditions. Doklady Akademii Nauk SSSR, 280(4), 796-798.

12. *Aliev F.A., Larin V.B.* Optimization of Linear Control System. Gordon and Breach publ., Amsterdam, 1998, 280 p.

13. *Aliev, F.A., Bordyug, B.A., Larin, V.B.* H2-optimization and the state space method in the synthesis of optimal controllers. Baku, Elm, 1991 (in Russian)

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OSİLYASİYA SİSTEMLƏRİNDƏ KƏSR TƏRTİB TÖRƏMƏNİN TƏYİNİ ÜÇÜN YENİ TƏRS MƏSƏLƏ

F.Ə.Əliyev, N.Ə.Əliyev

İlk dəfə olaraq osilyator sistemlərində tabeli həddə daxil olan kəsr tərtib törəmənin tənliyi üçün yeni tərs məsələ qoyulub. Uyğun koordinatların son qiymətlərində statistik verilənlər və ən kiçik kvadratlar üsulu vasitəsilə yuxarıda adı gedən parametrin təyini üçün üsul təklif olunub. Nəticələr ədədi üsulla illüstrasiya olunub.

Açar sözlər: *osilyasiya prosesi, tərs məsələ, kəsr tərtib törəmə, statistik verilənlər, ən kiçik kvadratlar üsulu*

НОВАЯ ОБРАТНАЯ ЗАДАЧА ДЛЯ ОПРЕДЕЛЕНИЯ ПОРЯДКА ДРОБНЫХ ПРОИЗВОДНЫХ КОЛЕБАТЕЛЬНОЙ СИСТЕМЫ

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Впервые дана постановка новых обратных задач для определения порядка дробных производных в подчиненных членах колебательного процесса. Предложен метод определения вышеуказанного параметра на основе статистических данных конечных значений соответствующей координаты и метода наименьших квадратов. Результат иллюстрируется числовым примером.

Ключевые слова: *колебательный процесс, обратная задача, дробная производная, статистические данные, метод наименьших квадратов*