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Informatics**PARALLEL SOLUTION METHODS IN OPTIMIZATION
OF GAS-LIFT PROCESS*****M.M.Hashemi¹, N.A.Ismailov², S.A. Ahari³**

(Presented by the academician of NAS of Azerbaijan F.A.Aliev)

In this paper, we investigated the use of parallel methods for solving ordinary differential equations, which describes the process of gas lift in oil extraction industries. A mathematical model based on this problem constructed. This research is related to an optimization problem. We solved this problem through Runge Kutta method using parallel algorithms. By using parallel technologies for solving this problem more efficiency achieved that the traditional algorithm. Parallel computing technologies have improved accuracy too.

Keywords: parallel computation, gas lift process, ordinary differential equation, Runge Kutta method

Introduction

Recently, in some of actual and vital problems we can find special characters that allow us to apply some modifications for achieving better result in comparison with current traditional solutions. In this paper our main focus is on the parallelization of Gas-lift process which is one of oil extraction methods. There are some problems arises from the fields of science and engineering represented by mathematical models. For example, oil extraction industry and other areas of application. These problems and applications can be written in the form of mathematical models like differential equations, either as a first order or higher order and linear or nonlinear ODEs. This study considers solving special ODE based on Initial Value Problems (IVP)s in forth order PIRK method. There are many scientific works focused on mathematical solving for oil extraction problems and specially related to gas-lift method. For more information you can see [1, 2]. But in fact, there wasn't a concrete and comprehensive mathematical model for mentioned problem and even existing methods and algorithms can't reach to acceptable accuracy and fastness. In our research we are focusing the results arises from [3-5]. This is

may be because the solution is needed in real time, as is the case for oil extraction or control and optimization systems applied in any field of science and engineering, or it is may be because computation time on a conventional sequential machine is so long that it adversely affects the productivity of engineers and scientists working on the design of complex systems, for example.

Exploring of researched problem. As mentioned before, the problems of motion of fluids, gases and gas-liquid mixtures in pipes related to gas-lift oil recovery are mathematically formulated as ODEs. The optimal control problems are posed based on the proposed models and some real assumptions. These problems can be used to design programmed paths and controls which underlie the controllers that stabilize the pressure or volume of injected gas for oil extraction. The theoretical performance analysis and a runtime simulation which executed on the special IBM multiprocessor system allowed us to evaluate the presented algorithm [6]. Obtained charts and numerical results proofed that our suggested algorithm works properly in comparison with the reached numerical results from non-parallel version. Figure 1

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gives adequate and subjective information about the gas lift problem.

The ODEs extracted from mentioned gas-lift problem which we want to do it in parallel is as below:

$$Q' = \frac{2a_1\rho_1 F_1 Q^2}{c_1^2 \rho_1^2 F_1^2 - Q^2}, \quad x \in [0, l-0] Q(0) = Q_0 \quad (2)$$

$$Q' = \frac{2a_2\rho_2 F_2 Q^2}{c_2^2 \rho_2^2 F_2^2 - Q^2}, \quad x \in [l+0, 2l]$$

$$Q(l+0) = \gamma Q(l-0) + (-\delta_3(Q(l-0) - \delta_2)^2 + \delta_1)\bar{Q}.$$

Where coefficients used in the equation are as:

$$a_1 = 0.1008449644636985 \quad \rho_1 = 0.717, \\ f_1 = 0.006021647718768237 \quad C_1 = 331,$$

$$a_2 = -89.77276884123468 \quad \rho_2 = 700, \\ f_2 = 0.004185386812745002 \quad C_2 = 850.$$

$$\gamma = 1, \quad \delta_1 = 0.1, \quad \delta_2 = 0.1, \quad \delta_3 = 0.02 \text{ and } \bar{Q} = 100.$$

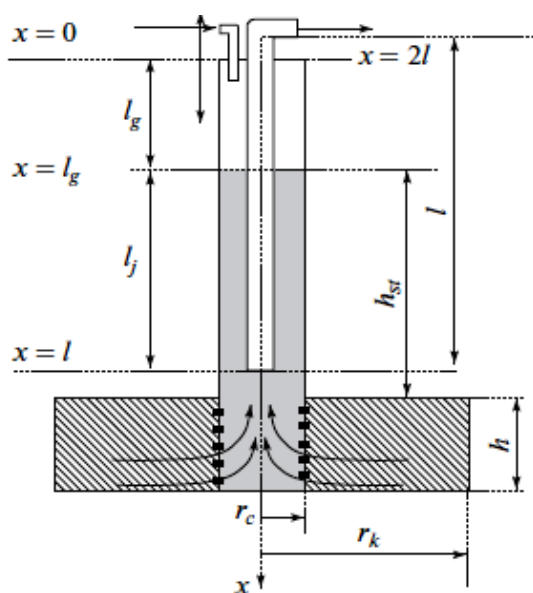
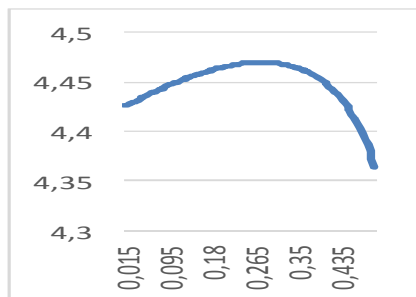
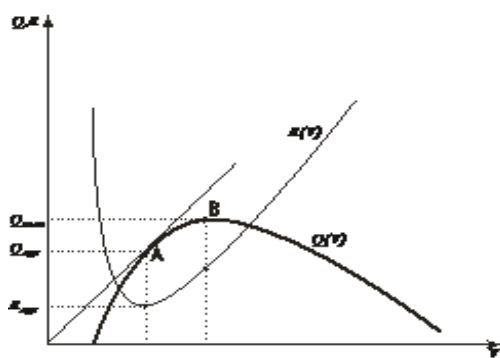


Fig 1. Gas lift system applied on an oil well

For more information about sequential and parallel solution methods of IRK you can see [7-10]. For designing parallel algorithm for solution of system (2) we use one of parallel computing technologies that named as Thread Computing method. By using thread computing method we can create some threads according to the number of physical processors or cores in multi processor system. The first created thread in the designed algorithm will execute solving operation process of equation with first amount of $Q(0)$ from given interval. At the same time in other created threads on the next amounts of $Q(0)$ from interval will be executed in parallel. This structure will be continued until finishing all determined amounts from given interval for $Q(0)$ s.

About optimization aspect of problem using parallel solution method. We would like to test suggested algorithm for vary large amounts of initial values from given interval and want to get $Q(2l)$ s as much as possible in order to increase accuracy of solution. General purpose in mathematical modeling of gas-lift system applied on an oil well is extracting oil from well by injection of gas into it. But this system aims to know what amount of injected gas can lift maximum amount of product from the well. To achieve this purpose we should solve system (2) by large amount of initial values to get the largest amount for $Q(2l)$ for increasing accuracy. By applying parallel algorithm for solving this system, we can test the equation for much more initial values in less execution time and as a result we can find the optimal value for $Q(0)$ and $Q(2l)$ just by using a simple comparison at the end of each execution.

The explained method can be implemented for many situations in any field of science and engineering containing ODEs related or not related to oil extraction problems. in both multi-processor and multicomputer architectures of multi processor environment. Obtained results by applying traditional numerical IRK method and by applying our parallel algorithm for solving (2) shown below as fig. 2 and 3.

Fig 2. Dependency of $Q(0)$ in sequentialFig 3. Dependency of $Q(0)$ in parallel

Focusing on figure 2 and figure 3, it clearly can be seen that how the system (2) reacts for initial values applying parallel and se-

quential (traditional) algorithms and programs. Also, the obtained results are same in both traditional non-parallel and parallel conditions. It means that our suggested algorithm works properly according to the reached charts. Also the maximum amount for $Q(2l)$ as the result we are searching is seems from fig. 2 and fig. 3 clearly too.

At the end, we would like to discuss about some numerical results of a real example from this computation in traditional non parallel and in parallel forms in table 1 and 2. Of course, here we just brought numerical result for a little set of numbers from given interval as a sample. We believe that it is enough for comparison.

These tables show that our suggested algorithm works properly according to the reached numerical results from non-parallel version. But here is an interesting point too; considering table 1 and table 2 we can easily find that in addition to fastness of suggested algorithm, the parallel solution version for mentioned problem has much more accuracy when comparing by sequential algorithm.

Table 1

Numerical results gotten from sequential algorithm

$Q(0)$	0.01	0.11	0.21	0.26	0.31	0.41	0.51
$Q(l-0)$	0.0164	0.11826	0.243	0.31359	0.3914	0.5797	0.8651
$Q(l+0)$	9.9939	10.118	10.202	10.222	10.222	10.119	9.6942
$Q(2l)$	4.4327	4.4569	4.4732	4.4771	4.4769	4.4572	4.3727

Table 2

Numerical results gotten from parallel solution method

$Q(0)$	$Q(l-0)$	$Q(l+0)$	$Q(2l)$
0.01	0.010063723685305	10.2223810465939	4.42480158959619
0.11	0.118291829545694	10.1176226474894	4.44889455491464
0.21	0.243162475262978	10.2021714866161	4.46516833952927
0.26	0.313813307652505	10.2223810465939	4.46903580408768
0.31	0.391762268746577	10.2215118258183	4.46886964006538
0.41	0.580635647577958	10.1186143961326	4.44908632760122
0.51	0.867815249268826	9.68873473524933	4.36393954214418

Conclusion

We proofed that, the parallel solution for solving and optimizing of gas lift problem has more accuracy and speed. This is because of computing the extracted equation for the large number of initial values from given interval. Also, we can change the factor of accuracy through the method RK. In non-parallel environment that we have just on processor or maximum 4 cores in present-day computers, reduction of this factor in IRK method will cause long execution time. But in parallel systems we can easily change this factor for achieving more accurate answers.

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QAZ LİFT MƏSƏLƏSİNİN OPTİMALLAŞDIRILMASINDA PARALEL HƏLL ÜSULLARI

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Bu məqalədə, neftçıxarma sənayesində qaz lift prosesini təsvir edən adi diferensial tənliklərin həlli üçün paralel metodların istifadəsi araşdırılmışdır. Bu məsələ əsasında riyazi model qurulmuşdur. Baxılan iş bir optimizasiya məsələsinə aiddir. Biz bu məsələni Runge-Kutta metodundan istifadə edərək paralel üsullarla həll etmişik. Ənənəvi üsullarla müqayisədə paralel hesablama texnologiyası daha effektiv olmuş və daha yüksək dəqiqlik əldə edilmişdir.

Açar sözlər: paralel hesablama, qaz lift prosesi, adi diferensial tənlik, Runge-Kutta metodu

ПАРАЛЛЕЛЬНЫЕ МЕТОДЫ РЕШЕНИЯ В ОПТИМИЗАЦИИ ПРОЦЕССА ГАЗЛИФТА

М.М.Хашеми, Н.А.Исмаилов, Ш.А.Ахари

В статье рассмотрены применения параллельных методов решения обыкновенных дифференциальных уравнений, которые описывают газлифтный процесс в нефтедобыче. Исследуется математическая модель, построенная на основе этой проблемы. Задача оптимизации решена методом Рунге-Кутты с применением параллельного вычисления. Использование параллельных алгоритмов для решения этой задачи, является более эффективным, чем традиционные алгоритмы, которые выполняются последовательно. Показано, что параллельные технологии вычислений улучшили точность.

Ключевые слова: параллельные вычисления, процесс газлифта, обыкновенное дифференциальное уравнение, метод Рунге-Кутты