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DIGITAL ALGORITHMS FOR CALCULATING THE DIFFERENTIAL FUNCTION OF NORMAL DISTRIBUTION OF NOISE

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The authors develop digital algorithms for calculating the differential function of noise distribution, discussing the case when the noise obeys the normal distribution law.

Keywords: noise, normal distribution law, differential distribution function, digital algorithms

Introduction

It is known that noise, which is an extraneous disturbance in systems of tracking, monitoring, control, diagnostics, forecasting, identification, management, etc., obstructs correct reception of the main useful signal. Noise masks and distorts the useful signal, impeding its detection. Therefore, the most important indicator of successful functioning of said systems is their noise resistance, by which we understand the ability of a system to resist obstructive effects of noise. There are many noise control methods, such as shielding, grounding, balancing, filtering, isolation, separation and orientation, circuit impedance level control, cable design, cancellation (frequency or time domain), etc. [1].

However, noises can be caused by various factors. Distortions of useful signals induced by such destabilizing factors as defects, wear and tear, corrosion, cracks, breaks and other malfunction of equipment, structure, engine, mechanism, etc. are also classified as noises [2]. In that case, not suppression of noise but its separation from the total of the registered signals and calculation of its probabilistic characteristics will allow one to obtain useful information on the nature of the noise and detect the initial stage of defect origin.

The additive noise $E(t)$ that emerges due to defect origin is superimposed on the useful signal $X(t)$:

$$G(t) = X(t) + E(t). \quad (1)$$

To carry out quantitative calculations of the effects of noise on the signal, one needs to know the main probabilistic characteristics of noise [3, 19-27; 44-55]. The authors of [4-6] developed methods for calculating some of the characteristics of noise of noisy signals, such as variance, mean square deviation. However, this information alone is not sufficient for solving the described problems, as such main characteristic of the random noise as the differential function of distribution remains unknown. Proposed below are digital algorithms for calculating the differential function of distribution of noise of the noisy signal $G(t)$ for the case when $E(t)$ obeys the normal law.

Problem statement. Let us assume that the stationary ergodic random signal $G(t)$ with the distribution law $f(g)$ consists of the sum of the stationary ergodic random main signal $X(t)$ with the arbitrary distribution law $f(x)$ and the stationary ergodic random interfering signal $E(t)$ with a zero mathematical expectation $m_e = 0$ and the normal distribution law, i.e. $G(t)$ can be represented as (1).

The distribution law $f(g)$ of the noisy signal $G(t)$ can always be determined based on the measure of agreement between the theoretical and statistical distributions [7, 149]. But in that case, the differential function of distribution $f(x)$ of the useful signal $X(t)$ cannot be separated from the differential function of distribution $f(\varepsilon)$ of the noise $E(t)$. At the same time, the pattern and variation dynamics of the

differential function of distribution $f(\varepsilon)$ of the noise $E(t)$ can be indicative of the changes in the technical condition of the object under investigation, i.e. one could extract the necessary useful information from them.

In systems of tracking, monitoring, control, diagnostics, forecasting, identification, management, etc., noise complies with the normal distribution law with a zero mean, i.e. is a Gaussian process at the initial stage of origin of defects, faults, malfunction, etc. Such noises have very low probability of their pulse spike amplitude exceeding the tripled mean square value. Such noise emerges when independent white noises are added together and it is common in practical problems. The differential function of distribution of that noise is determined from the expression [3, 20-21]:

$$N(\varepsilon, m_\varepsilon, \sigma_\varepsilon) = \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} e^{-\frac{(\varepsilon - m_\varepsilon)^2}{2\sigma_\varepsilon^2}},$$

where σ_ε , m_ε are the mean square deviation and the mathematical expectation of the noise $E(t)$.

As the stationary random noise $E(t)$ is ergodic, then its mathematical expectation m_ε and the mean square deviation σ_ε have the same value for each of the random functions in the set. Therefore, let us represent the differential function of the normal distribution of the noise as follows:

$$N(\varepsilon, m_\varepsilon, \sigma_\varepsilon) = N(\varepsilon),$$

$$N(\varepsilon) = \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} e^{-\frac{(\varepsilon - m_\varepsilon)^2}{2\sigma_\varepsilon^2}}. \quad (2)$$

However, as the degree of malfunction increases in course of time, the shape of the differential curve of the normal distribution changes first; then, as the defect takes an explicit form, the distribution law itself changes from the normal one. Therefore, the problem of determining the differential function of normal

distribution $N(\varepsilon)$ of the noise $E(t)$ arises. This allows for identifying the dynamics of the variation of the shape of the normal curve $N(\varepsilon)$ of the noise $E(t)$ in time, determining the probability with which it takes the maximum value and the most probable values within the interval $-\sigma_\varepsilon \leq E(t) \leq \sigma_\varepsilon$.

If we build a matrix of informative attributes, with the listed characteristics being the elements of that matrix, we can use the combinations of their values to determine the initial period of defect origin and the moments when maintenance, regular and overhaul repairs are due. Therefore, in the following paragraphs, we propose digital algorithms for determining the differential function of normal distribution $N(\varepsilon)$ of the noise $E(t)$ of the noisy signal $G(t)$, which are easy to use in practice.

Digital algorithms for determining the differential function of normal distribution of the noise. We know that the differential function of normal distribution $N(\varepsilon)$ of the noise $E(t)$ is characterized by two parameters: the mathematical expectation m_ε and the mean square deviation $\sigma_\varepsilon = \sqrt{D_\varepsilon}$. As the noise has a zero mean $m_\varepsilon = 0$, then the problem amounts to the calculation of the parameter σ_ε alone.

Let us assume that the additive noisy digital signal $G(\Delta t)$ consisting of the useful signal $X(\Delta t)$ and the noise $E(\Delta t)$ comes from the sensor, which is placed in the area of effect of the factors affecting the object and receives digital information from that object. The signal $G(\Delta t)$ is sampled at the interval Δt . Let us use the following expression for calculating the correlation function $R_{gg}(\mu)$ of the noisy signal [7-8]:

$$R_{GG}(\mu) = \frac{1}{N} \sum_{i=1}^N \dot{G}(i\Delta t) \dot{G}((i+\mu)\Delta t) =$$

$$= \frac{1}{N} \sum_{i=1}^N \dot{X}(i\Delta t) \dot{X}((i+\mu)\Delta t) +$$

$$\begin{aligned}
& + \frac{1}{N} \sum_{i=1}^N \dot{X}(i\Delta t) \dot{E}((i+\mu)\Delta t) + \\
& + \frac{1}{N} \sum_{i=1}^N \dot{E}(i\Delta t) \dot{X}((i+\mu)\Delta t) + \\
& + \frac{1}{N} \sum_{i=1}^N \dot{E}(i\Delta t) \dot{E}((i+\mu)\Delta t), \quad (3)
\end{aligned}$$

where the centered values are calculated from the expressions: $\dot{G}(t) = G(t) - m_g$, $\dot{X}(t) = X(t) - m_x$; m_g , m_x are the mathematical expectations of $G(t)$ and $X(t)$, respectively.

Considering that the useful signal $X(t)$ and the noise $E(t)$ are non-correlated, i.e.

$$\begin{aligned}
& \frac{1}{N} \sum_{i=1}^N \dot{X}(i\Delta t) \dot{E}((i+\mu)\Delta t) = 0, \\
& \frac{1}{N} \sum_{i=1}^N \dot{E}(i\Delta t) \dot{X}((i+\mu)\Delta t) = 0, \quad (4)
\end{aligned}$$

we can write the following:

$$\begin{aligned}
R_{GG}(\mu) &= \frac{1}{N} \sum_{i=1}^N \dot{G}(i\Delta t) \dot{G}((i+\mu)\Delta t) = \\
&= \frac{1}{N} \sum_{i=1}^N \dot{X}(i\Delta t) \dot{X}((i+\mu)\Delta t) + \\
&+ \frac{1}{N} \sum_{i=1}^N \dot{E}(i\Delta t) \dot{E}((i+\mu)\Delta t) = \\
&= R_{XX}(\mu) + R_{\varepsilon\varepsilon}(\mu). \quad (5)
\end{aligned}$$

Thus, the correlation function $R_{GG}(\mu)$ of the noisy signal $G(t)$ consists of the sum of the correlation functions $R_{XX}(\mu)$ and $R_{\varepsilon\varepsilon}(\mu)$ of the useful signal $X(t)$ and the noise $E(t)$, respectively.

In practice, for such infra-low frequency slow-flowing technological processes as oil refinery, petrochemistry, when $\mu = \Delta t$ is significantly (multifold) lower compared to the observation time T , the noise $E(t)$ forms from high frequency spectra due to such faults as wear and tear, corrosion, carbon formation, etc. and has a higher spectrum than the useful component $X(t)$ itself. The meaning of the useful component has no time to change in the interval Δt , and $X(t + \Delta t)$ matches the value of $X(t)$, i.e.

$$X(t + \Delta t) = X(t). \quad (6)$$

This equality holds for the cases when T is, for instance, 30-50 hours, and Δt is seconds or minutes (depending on the specifics of the process under investigation). In that case, the sampling interval Δt is selected based on the spectrum of the noise $E(t)$ and not of the useful component $X(t)$, i.e. $\Delta t = 1/2\omega_\varepsilon$, where ω_ε is the noise cutoff frequency.

Obviously, such a strict equality does not hold for all real-life processes, but for processes of oil refinery, petrochemistry. For other technological processes, an approximate equation is acceptable. Then for said facilities, when condition (6) holds, the relation $\frac{R_{XX}(\mu = \Delta t)}{R_{XX}(0)} = 1$, i.e. [8]:

$$R_{XX}(\mu = \Delta t) = R_{XX}(0) \quad (7)$$

At the same time, since the random noise $E(t)$ emerges when independent white noises are added together, it has the correlation time of $\mu = 0$, and the correlation function $R_{\varepsilon\varepsilon}(\mu)$ is a δ -function [8], i.e.

$$R_{\varepsilon\varepsilon}(\mu) = \begin{cases} R_{\varepsilon\varepsilon}(\mu = 0) & \text{at } \mu = 0 \\ 0 & \text{at } \mu \neq 0 \end{cases} \quad (8)$$

Considering that the estimates of autocor-

relation functions of the useful signal $X(t)$ and the noise $E(t)$ at a zero time shift $\mu=0$ are the variances of the useful signal $X(t)$ and the noise $E(t)$, respectively:

$$R_{XX}(\mu=0)=D_X, R_{EE}(\mu=0)=D_E, \quad (9)$$

in the sufficiently small, compared to the observation time T , interval $\mu=\Delta t$, with regard to expressions (5)-(9), we get:

$$R_{GG}(\mu=0)-R_{GG}(\mu=\Delta t)=R_{\varepsilon\varepsilon}(\mu=0).$$

Then the noise variance is calculated from the expression:

$$D_\varepsilon^*=R_{GG}(\mu=0)-R_{GG}(\mu=\Delta t). \quad (10)$$

Therefore, the mean square deviation σ_ε^* of the noise $E(t)$ can be calculated from the expression:

$$\sigma_\varepsilon^*=\sqrt{D_\varepsilon^*}=\sqrt{R_{GG}(\mu=0)-R_{GG}(\mu=\Delta t)}. \quad (11)$$

Thus, the parameter σ_ε^* of the normal differential distribution of the noise $E(t)$ can be calculated by determining the square root from the difference of the estimates of the autocorrelation correlation function $R_{GG}(\mu)$ of the noisy signal at a zero $\mu=0$ and unit $\mu=\Delta t$ time shifts.

Then the differential function of the normal distribution $N(\varepsilon, m_\varepsilon, \sigma_\varepsilon)$ of the noise $E(t)$ of the noisy signal $G(t)$ with the mathematical expectation $m_\varepsilon=0$, with provision for formula (2), can be found, using the expression:

$$N^*(\varepsilon)=\frac{1}{\sigma_\varepsilon^*\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2(\sigma_\varepsilon^*)^2}}. \quad (12)$$

Considering that the absolute value deviation from the mathematical expectation for the normally distributed process does not exceed the

tripled mean square deviation, the discrete values of the differential function of distribution $N^*(\varepsilon)$ of the noise $E(t)$ are calculated within the interval $-3\sigma_\varepsilon^*\leq E(t)\leq 3\sigma_\varepsilon^*$. To that end, we:

– calculate the minimum and maximum values of $E(t)$: $\varepsilon_{\min}=-3\sigma_\varepsilon^*$; $\varepsilon_{\max}=3\sigma_\varepsilon^*$;

– set a sequence of possible values of $E(t)$ in ascending order from $\varepsilon_{\min}$ to $\varepsilon_{\max}$ at a certain interval $\Delta\varepsilon$: $\varepsilon(1)=\varepsilon_{\min}$, $\varepsilon(i+1)=\varepsilon(i)+\Delta\varepsilon$, ..., $\varepsilon_{\max}$;

– form a sequence of possible values of the noise $\varepsilon(1)$, $\varepsilon(2)$, $\varepsilon(3)$, $\varepsilon(4)$, ..., $\varepsilon_{\max}$, for which the condition $\varepsilon(i-1)<\varepsilon(i)$ holds;

– calculate the differential function of the normal distribution within the interval $-3\sigma_\varepsilon^*\leq E(t)\leq 3\sigma_\varepsilon^*$ in points $\varepsilon(1)$, $\varepsilon(2)$, $\varepsilon(3)$, $\varepsilon(4)$, ..., $\varepsilon_{\max}$, using one of the formulas:

$$N^*(\varepsilon(i))=\frac{1}{\sigma_\varepsilon^*\sqrt{2\pi}}e^{-\frac{(\varepsilon(i))^2}{2(\sigma_\varepsilon^*)^2}}, \quad (13)$$

$$N^*(\varepsilon(i))=\frac{1}{\sqrt{2\pi\cdot(R_{GG}(\mu=0)-R_{GG}(\mu=\Delta t))}}\times e^{-\frac{(\varepsilon(i))^2}{2(R_{GG}(\mu=0)-R_{GG}(\mu=\Delta t))}}. \quad (14)$$

Conclusions

The developed digital algorithms can be used in systems of tracking, monitoring, control, diagnostics, forecasting, identification, management, etc. for timely detection of incipient defects and determining the time for maintenance, regular and overhaul repairs.

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KÜYÜN DİFERENSİAL NORMAL PAYLANMA FUNKSİYASININ HESABLANMASININ ƏDƏDİ ALQORİTMLƏRİ

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Küyün diferensial paylanma funksiyasının hesablanması ədədi alqoritmləri işlənmişdir. Küyün normal paylanmaya malik olan hala baxılmışdır.

Açar sözlər: küy, normal paylanma qanunu, diferensial paylanma funksiyası, ədədi alqoritmlər

ЦИФРОВЫЕ АЛГОРИТМЫ ВЫЧИСЛЕНИЯ ДИФФЕРЕНЦИАЛЬНОЙ ФУНКЦИИ НОРМАЛЬНОГО РАСПРЕДЕЛЕНИЯ ПОМЕХИ

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Разработаны цифровые алгоритмы вычисления дифференциальной функции распределения помехи зашумленного сигнала. Рассмотрен случай, когда помеха подчиняется нормальному закону распределения.

Ключевые слова: помеха, нормальный закон распределения, дифференциальная функция распределения, цифровые алгоритмы