

UDC 519.216

ROBUST TECHNOLOGY FOR FORMING CORRELATION MATRICES OF MATHEMATICAL MODELS OF THE DYNAMICS OF CONTROL OBJECTS

Academician of ANAS T.A.Aliev, N.F.Musayeva, U.E.Sattarova, N.E.Rzayeva

The authors analyze the challenges of forming correlation matrices in solving problems of identification of matrix models of the dynamics for real-life industrial objects. They propose generalized algorithms that allow for reducing those matrices to similar matrices of useful signals. The specific characteristics of real-life noisy technological parameters are taken into account and it is also demonstrated that said algorithms can be used in the absence of a correlation between the useful signal and the noise and in the presence of such correlation.

Keywords: stochastic process, identification, technological parameter, noise, noisy signal, correlation function, correlation matrices, normalized estimates, dynamics models

Introduction

It is known [1-6] that one of the main challenges in solving problems of automated control of industrial objects is establishing the quantitative interrelations between technological parameters characterizing the processes in those objects both in statics and dynamics. The statistical correlation method for determining these dynamic characteristics is based on the solution of an integral equation that includes the correlation functions $R_{xx}(\tau)$ and $R_{xy}(\tau)$ of the input $X(\tau)$ and output $Y(\tau)$ signals. However, in real-life objects, interferences and noises are imposed upon the useful signal (that has to be obtained with the least possible amount of distortion), thus hindering the calculation of the estimates of their static characteristics. The paper considers one possible option of creating alternative digital methods and technologies for forming the correlation matrix of the dynamic model of industrial objects, which is generally described with matrix equations of the following type

$$\begin{aligned} \bar{R}_{xy}(\mu) &\approx \bar{R}_{xx}(\mu) \bar{W}(\mu), \\ \mu &= 0, \Delta t, 2\Delta t, \dots, (N-1)\Delta t, \end{aligned} \quad (1)$$

where $\bar{R}_{xx}(\mu)$ is the square symmetric matrix of the autocorrelation functions with dimension

$N \times N$ of the centered input signal $X(t)$; $\bar{R}_{xy}(\mu)$ is the column vector of the cross-correlation functions between the input $X(t)$ and the output $Y(t)$, $\bar{W}(\mu)$ is the column vector of the impulsive admittance functions. However, the real technological parameters $g(\Delta t)$ and $\eta(i\Delta t)$ are the sum of the useful signals $X(t)$, $Y(t)$ and the noises $\varepsilon(i\Delta t)$, $\eta(i\Delta t)$, therefore, matrix equation (1) of real-life technological processes can be represented in the following way:

$$\begin{aligned} \bar{R}_{g\eta}(\mu) &= \bar{R}_{gg}(\mu) \bar{W}(\mu), \\ \mu &= 0, \Delta t, 2\Delta t, \dots, (N-1)\Delta t \\ \bar{r}_{g\eta}(\mu) &\approx \bar{r}_{gg}(\mu) \bar{W}(\mu), \\ \mu &= 0, \Delta t, 2\Delta t, \dots, (N-1)\Delta t \end{aligned}$$

where $\bar{r}_{gg}(\mu)$ is the square symmetric matrix of the normalized autocorrelation functions with dimension $N \times N$ of the centered input signal $g(t)$; $\bar{r}_{g\eta}(\mu)$ is the column vector of the normalized cross-correlation functions between the input $g(t)$ and the output. However, obvious inequalities emerge in this case:

$$\left. \begin{aligned} \vec{R}_{XX}(\mu) &\neq \vec{R}_{gg}(\mu) \\ \vec{R}_{XY}(\mu) &\neq \vec{R}_{g\varphi}(\mu) \\ \vec{r}_{XX}(\mu) &\neq \vec{r}_{gg}(\mu) \\ \vec{r}_{XY}(\mu) &\neq \vec{r}_{g\varphi}(\mu) \end{aligned} \right\}. \quad (2)$$

As a result, in practice adequacy of identification of the model of the dynamics (1) of technological processes fails in many cases.

Problem statement

To ensure an adequate solution to the problem of identification of matrix models of the dynamics of industrial objects, we need to develop a robust technology for forming the correlation matrices $\vec{R}_{gg}^R(\mu)$, $\vec{R}_{g\eta}^R(\mu)$, $\vec{r}_{gg}^R(\mu)$, $\vec{r}_{g\eta}^R(\mu)$ that ensures that the following equalities hold:

$$\left. \begin{aligned} \vec{R}_{gg}^R(\mu) &\approx \vec{R}_{XX}(\mu) \\ \vec{R}_{g\eta}^R(\mu) &\approx \vec{R}_{XY}(\mu) \\ \vec{r}_{gg}^R(\mu) &\approx \vec{r}_{XX}(\mu) \\ \vec{r}_{g\eta}^R(\mu) &\approx \vec{r}_{XY}(\mu) \end{aligned} \right\} \quad (3)$$

Technology for forming the correlation matrix in the presence of a correlation between the useful signal and the noise. Technological parameters of real-life industrial objects are characterized by the presence of the noise $\varepsilon(t)$, which in most cases is in correlation with the useful signal $X(t)$ [2,3,8]. Therefore, the formula for determining the estimate of $R_{gg}(\mu)$ can be represented in the following form:

$$\begin{aligned} R_{gg}(\mu) &\approx \frac{1}{N} \sum_{i=1}^N g(i\Delta t)g((i+\mu)\Delta t) \approx \\ &\approx \begin{cases} R_{XX}(0) + D_\varepsilon & \text{at } \mu = 0 \\ R_{XX}(\mu) + R_{X\varepsilon}(\mu) & \text{at } \mu \neq 0. \end{cases} \end{aligned}$$

Thus, to make correlation matrices equivalent to the matrices of useful signals, we need to subtract the value of D_ε from the estimates of $R_{gg}(0)$, and the value of $R_{X\varepsilon}(\mu)$ from the estimates of $R_{gg}(\mu)$, i.e.

$$\begin{aligned} \vec{R}_{gg}^R(\mu) &\approx \\ &\approx \begin{vmatrix} R_{gg}(0) - D_\varepsilon & R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) & \dots & R_{gg}[(N-1)\Delta t] - R_{X\varepsilon}[(N-1)\Delta t] \\ R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) & R_{gg}(0) - D_\varepsilon & \dots & R_{gg}[(N-2)\Delta t] - R_{X\varepsilon}[(N-2)\Delta t] \\ \dots & \dots & \dots & \dots \\ R_{gg}[(N-1)\Delta t] - R_{X\varepsilon}[(N-1)\Delta t] & R_{gg}[(N-2)\Delta t] - R_{X\varepsilon}[(N-2)\Delta t] & \dots & R_{gg}(0) - D_\varepsilon \end{vmatrix} \\ \vec{r}_{gg}^R(\mu) &\approx \\ &\approx \begin{vmatrix} 1 & \frac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t)}{D_g - D_\varepsilon} & \dots & \frac{R_{gg}[(N-1)\Delta t] - R_{X\varepsilon}[(N-1)\Delta t]}{D_g - D_\varepsilon} \\ \frac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t)}{D_g - D_\varepsilon} & 1 & \dots & \frac{R_{gg}[(N-2)\Delta t] - R_{X\varepsilon}[(N-2)\Delta t]}{D_g - D_\varepsilon} \\ \dots & \dots & \dots & \dots \\ \frac{R_{gg}[(N-1)\Delta t] - R_{X\varepsilon}[(N-1)\Delta t]}{D_g - D_\varepsilon} & \frac{R_{gg}[(N-2)\Delta t] - R_{X\varepsilon}[(N-2)\Delta t]}{D_g - D_\varepsilon} & \dots & 1 \end{vmatrix} \end{aligned}$$

where D_ε is the noise variance, $R_{X\varepsilon}(\mu)$ is the cross-correlation function between $X(t)$ and $\varepsilon(t)$.

Given below are the formulas for determining the estimates of D_ε and $R_{X\varepsilon}(\mu)$.

$$\begin{aligned} D_\varepsilon &\approx \frac{1}{N} \sum_{i=1}^N \left[g(i\Delta t)g(i\Delta t) - 2g(i\Delta t)g((i+1)\Delta t) + \right. \\ &\quad \left. + g(i\Delta t)g((i+2)\Delta t) \right], \end{aligned} \quad (4)$$

$$R_{X\varepsilon}(\mu) \approx \frac{1}{2} R_{X\varepsilon}''(\mu) \quad (5)$$

where

$$\begin{aligned} R_{X\varepsilon}''(\mu) &\approx \frac{1}{N} \sum_{i=1}^N g(i\Delta t)[g((i+\mu+1)\Delta t) - \\ &\quad - g((i+\mu)\Delta t) - 3g((i+\mu+\lambda+1)\Delta t) + \\ &\quad + 2g((i+\mu+\lambda)\Delta t) + g((i+\mu+\lambda+2)\Delta t)] \end{aligned} \quad (6)$$

For instance, when $\lambda = 1$, i.e. $R_{X\varepsilon}(\Delta t) > 0$, $R_{X\varepsilon}(2\Delta t) \approx 0$, expression (6) will have the following form:

$$\begin{aligned} R_{X\varepsilon}''(\Delta t) &\approx \frac{1}{N} \sum_{i=1}^N g(i\Delta t)[g((i+1+1)\Delta t) - \\ &\quad - g((i+1)\Delta t) - 3g((i+1+1+1)\Delta t) + \\ &\quad + 2g((i+1+1)\Delta t) + g((i+1+1+2)\Delta t)] \end{aligned}$$

And the expression for determining $R_{X\varepsilon}(\Delta t)$ will have the following form:

$$R_{X\varepsilon}(\Delta t) \approx \frac{1}{2} R''_{X\varepsilon}(\Delta t).$$

Similarly, when $\lambda = 3$, i.e. $R_{X\varepsilon}(\Delta t) > 0$, $R_{X\varepsilon}(2\Delta t) > 0$, $R_{X\varepsilon}(3\Delta t) > 0$, $R_{X\varepsilon}(4\Delta t) \approx 0$, the expressions for determining $R''_{X\varepsilon}(\Delta t)$ and $R_{X\varepsilon}(\Delta t)$ will have the following form:

$$\begin{aligned} R''_{X\varepsilon}(\Delta t) &\approx \frac{1}{N} \sum_{i=1}^N g(i\Delta t) [g((i+1+1)\Delta t) - \\ &- g((i+1)\Delta t) - 3g((i+1+3+1)\Delta t) + \\ &+ 2g((i+1+3)\Delta t) + g((i+1+3+2)\Delta t)] \\ R_{X\varepsilon}(\Delta t) &\approx \frac{1}{2} R''_{X\varepsilon}(\Delta t). \end{aligned}$$

Our research has demonstrated that the estimates of $R_{X\varepsilon}(\mu)$ can also be determined by means of the cross-correlation functions $R_{X\varepsilon}^*(\mu)$. To that end, we determine the estimate of $R_{X\varepsilon}^*(\mu)$ by means of the generalized expression.

$$R_{X\varepsilon}^*(\mu) \approx \frac{1}{2} R'_{X\varepsilon}(\mu). \quad (7)$$

And the estimate $R'_{X\varepsilon}(\Delta t)$ is determined from the formula

$$\begin{aligned} R'_{X\varepsilon}(\mu) &\approx \frac{1}{N} \sum_{i=1}^N \text{sgn } g(i\Delta t) [g((i+\mu+1)\Delta t) - \\ &- g((i+\mu)\Delta t) - 3g((i+\mu+\lambda+1)\Delta t) + \\ &+ 2g((i+\mu+\lambda)\Delta t) + g((i+\mu+\lambda+2)\Delta t)] \quad (8) \end{aligned}$$

Thus, in the presence of a correlation between $X(t)$ and $\varepsilon(t)$ at $\mu=0$, $\mu=\Delta t$, $\mu=2\Delta t$, etc., the expression for determining the corresponding estimates $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$,... can be represented in the following form:

$$\left. \begin{aligned} R_{X\varepsilon}(0) &\approx \frac{\Delta R_{gg}(0) R_{X\varepsilon}^*(0)}{\Delta R_{gg}^*(0)} \\ R_{X\varepsilon}(\Delta t) &\approx \frac{\Delta R_{gg}(\Delta t) R_{X\varepsilon}^*(\Delta t)}{\Delta R_{gg}^*(\Delta t)} \\ R_{X\varepsilon}(2\Delta t) &\approx \frac{\Delta R_{gg}(2\Delta t) R_{X\varepsilon}^*(2\Delta t)}{\Delta R_{gg}^*(2\Delta t)} \\ &\dots\dots\dots \\ R_{X\varepsilon}(\mu) &\approx \frac{\Delta R_{gg}(\mu) R_{X\varepsilon}^*(\mu)}{\Delta R_{gg}^*(\mu)} \end{aligned} \right\} \quad (9)$$

The application of generalized expressions (5)-(8) opens the way by determining the estimates $R_{X\varepsilon}(0)$, $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, etc. to correct the corresponding elements of correlation matrices. Therefore, in the presence of a correlation between $X(t)$ and $\varepsilon(t)$, subtraction of the corresponding estimates $R_{X\varepsilon}(\Delta t)$, $R_{X\varepsilon}(2\Delta t)$, $R_{X\varepsilon}(3\Delta t)$, ... and the value of D_ε from the elements of the columns and lines of the correlation matrices that represent the estimates of $R_{gg}(\Delta t)$, $R_{gg}(2\Delta t)$, $R_{gg}(3\Delta t)$, ... leads to their correction. For instance, when $R_{X\varepsilon}(\Delta t) > 0$, $R_{X\varepsilon}(2\Delta t) \approx 0$, $R_{X\varepsilon}(3\Delta t) \approx 0$, ..., using the result obtained from expressions (5)-(8) the element of the second column in the first line and the second line in the first column ($R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) = R_{XX}(\Delta t)$) is corrected, and then the robust normalized correlation matrices of noisy signals will have the following form:

$$\overrightarrow{r}_{gg}(\mu) \approx \overrightarrow{r}_{XX}(\mu) \approx$$

$$\begin{vmatrix} 1 & \frac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t)}{D_g - D_\varepsilon} & \dots & \frac{R_{gg}[(N-1)\Delta t] \approx R_{XX}[(N-1)\Delta t]}{D_g - D_\varepsilon} \\ \frac{R_{gg}(\Delta t) - R_{X\varepsilon}(\Delta t) \approx R_{XX}(\Delta t)}{D_g - D_\varepsilon} & 1 & \dots & \frac{R_{gg}[(N-2)\Delta t] \approx R_{XX}[(N-2)\Delta t]}{D_g - D_\varepsilon} \\ \dots & \dots & \dots & \dots \\ \frac{R_{gg}[(N-1)\Delta t] \approx R_{XX}[(N-1)\Delta t]}{D_g - D_\varepsilon} & \frac{R_{gg}[(N-2)\Delta t] \approx R_{XX}[(N-2)\Delta t]}{D_g - D_\varepsilon} & \dots & 1 \end{vmatrix} \quad (10)$$

In that case, the result of formation of correlation matrices is regarded as valid only when the estimates of $R_{X\varepsilon}(\mu)$ at

$\mu = 0, \mu = \Delta t, \mu = 2\Delta t, \mu = 3\Delta t \dots$ obtained from expressions (5)-(8) match, i.e. the adequacy of the obtained results is achieved through their duplication. Therefore, after such correction, the obtained matrix can be considered equivalent to the matrix of useful signals.

Conclusion

The authors propose two alternative robust generalized technologies that enable one to reduce the correlation matrices of noisy technological processes to the matrices of their useful signals both in the absence of a correlation between the useful signal and the noise and in the presence of such. The adequacy of the results is achieved through duplicating the obtained estimates of the elements of matrices by both methods.

REFERENCES

1. Солодолников В.В. Введение в статистическую динамику систем автоматического управления. М.: Государственное изд-во технико-теоретической литературы, 1952, 163 с.
2. Telman Aliev, Robust Technology with Analysis of Interference in Signal Processing, Kluwer Academic/Plenum Publishers, New York (2003), 199 p.
3. Telman Aliev, Digital Noise Monitoring of Defect Origin, Springer, London (2007), 235p.
4. Алиев Т.А., Мусаева Н. Ф. Алгоритм уменьшения погрешностей оценки корреляционной функции сигнала с шумом// РАН СО «Автометрия», №4, 1995, с.105-112.
5. Aliev T.A., Musaeva N.F. An algorithm for Eliminating Mikroerrors of noise in the solution of Statistical Dynamics problems// Automation and Remote Control Vol 59. (Plenum publishing corporation), №5,1998, p.679-688.
6. Алиев Т.А., Амиров З.А. Алгоритм выбора параметров регуляризации при статистической идентификации// РАН, Автоматика и телемеханика, №6, 1998, с.130-139.
7. Aliev T.A., Musaeva N.F., Sattarova.U. E. Technology of calculating robust normalized correlation matrices// Cybernetics and Systems Analysis, Springer, No.1, Vol 47, 2011, pp.152-165.
8. Aliev T.A., Guluyev G.A., Pashayev F.H, Sadygov A.B. Noise monitoring technology for objects in transition to the emergency state// Mechanical Systems and Signal Processing, Elsevier. (Amsterdam), 2012, vol.27, p.755-762.
9. Aliev T.A., Abbasov A.M., Guluyev G.A., Pashaev F.H., Sattarova U.E. System of robust noise monitoring of anomalous seismic processes// Soil Dynamics and Earthquake Engineering, Vol. 32, Issue 1, 2013, p. 11–25.
10. Алиев Т.А., Мусаева Н.Ф., Сулейманова М.Т., Газызаде Б.И. Аналитическое представление функции плотности нормального распределения шума// Проблемы управления и информатики, 2015, N4, с. 104-118.

Institute of Control Systems of ANAS
telmancyber@rambler.ru

OBJEKTŁƏRİN STATİSTİK DİNAMİK MODELŁƏRİNİN İDENTİFİKASİYASINI TƏMİN EDƏN ROBAST TEXNOLOGİYALARIN NƏZƏRİ ƏSASLARI

T.A.Əliyev, N.F.Musayeva, Ü.E.Səttarova, N.E.Rzayeva

Real istehsalat obyektlərinin dinamik modellərinin korrelyasiya matrislərinin tərtib edilməsində qarşıya çıxan çətinliklər analiz olunmuşdur. Küylənmiş texnoloji parametrlərin matrislərinin onların faydalı siqnallarının matrislərinə gətirilməsinin alqoritmləri təklif edilmişdir. Alınan nəticələrin adekvatlığını təmin etmək üçün real kəsilməz texnoloji proseslərin xüsusiyyətləri nəzərə alınmış və təklif olunan texnologiyaların faydalı siqnalla küy arasında həm korrelyasiya olan hal üçün, həm də korrelyasiya olmayan hal üçün effektivliyi göstərilmişdir.

Açar sözlər: stoxastik proses, identifikasiya, texnoloji parametr, küy, küylənmiş siqnal, korrelyasiya funksiyası, korrelyasiya matrisləri, normallaşmış qiymətlər, dinamika modelləri

**РОБАСТНАЯ ТЕХНОЛОГИЯ ФОРМИРОВАНИЯ КОРРЕЛЯЦИОННЫХ МАТРИЦ
МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ ДИНАМИКИ ОБЪЕКТОВ УПРАВЛЕНИЯ****Т.А.Алиев, Н.Ф.Мусаева, У.Э.Саттарова, Н.Э.Рзаева**

Анализируются трудности формирования корреляционных матриц при решении задач идентификации матричных моделей динамики реальных производственных объектов. Предложены обобщенные алгоритмы, позволяющие свести эти матрицы к матрицам, аналогичным матрицам полезных сигналов. При этом учтены специфики реальных зашумленных технологических параметров и показана возможность применения данных алгоритмов как для случая, когда между полезным сигналом и помехой нет корреляции, так и для случая, когда корреляция между ними присутствует.

Ключевые слова: *стохастический процесс, идентификация, технологический параметр, помеха, зашумленный сигнал, корреляционная функция, корреляционные матрицы, нормированные оценки, модели динамики*