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STRESSED STATE AND LONGEVITY OF CORRODED WEDGE WITH CONCENTRATED FORCE APPLIED IN THE DIRECTION PERPENDICULAR TO SYMMETRY AXIS

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A problem of the action of concentrated force on an isotropic corroded wedge, is solved. The concentrated force is applied to the vertex of the wedge in the direction perpendicular to the symmetry axis. By solving the problem, the value of time corresponding to longevity of the wedge under corrosive wear, is determined. The condition for the value of the applied concentrated force at which there exists critical value of longevity time, is found.

Keywords: wedge, concentrated force, corrosive wear, longevity

Introduction

Prognostication of longevity of industrial constructions is a major problem in designing and operation of machines and mechanisms. Definition of stress-strain state of a construction with regard to action of environment is on the basis of this problem. The construction's material is corroded under the action of environment and this reduces to change of geometrical sizes of the construction, to redistribution of its stress-strain state. Definition of stress-strain state is necessary for true estimation of the limit state of the construction and this state depends on the applied external load and property of environment.

In the paper we consider deformation of a plate in the form of a wedge under the action of concentrated force applied to the vertex in the direction perpendicular to the symmetry axis. The wedge is subjected to corrosion uniformly distributed on the surface. Such constructions are encountered in aircraft industry, in transport, in maritime structures, etc. The solved problem is of practical application.

The goal of the paper is to find the values of time under which the considered wedge preserves its serviceability. Furthermore, the condition on the value of the applied force subject to which the limit value of longevity exists, is determined.

Problem statement. Let us consider a plate in the form of wedge made of isotropic-

elastic material: Opening of the wedge is 2α (fig.). At the vertex of the wedge, the concentrated force is applied in the direction perpendicular to the symmetry axis. Let the plate under consideration be subjected to corrosion uniformly distributed along the surface. The plate's material is corroded. We will assume that the plate's edges are corrosion-proof. Furthermore, we assume that in the corrosion process only the plate's thickness changes the would-be change of the thickness doesn't reduce to change of configuration of faces.

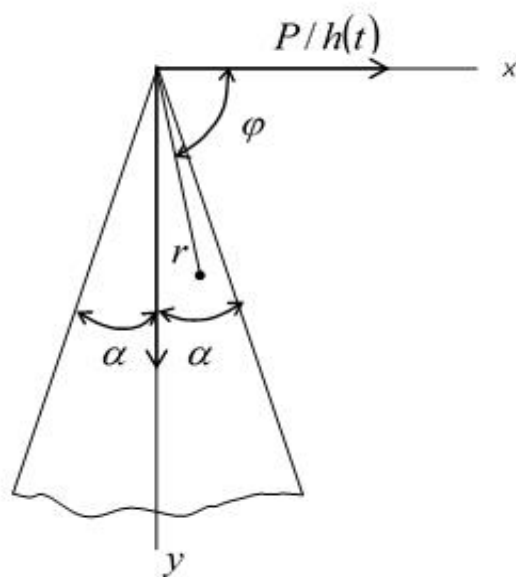


Fig. A wedge under the action of concentrated force

We use polar system of coordinates (r, φ) (fig.). Let $P = \text{const}$ be a concentrated force per a unit thickness of a wedge-shaped plate, h be the plate's thickness: $h = h(t)$, where t is time. In the corrosion process the thickness h diminishes: $dh(t)/dt < 0$. We assume that at the vertex of the wedge-shaped plate, the concentrated force $P/h(t)$, where $P = \text{const}$, acts in the direction perpendicular to the symmetry axis. In this case in the plate the plane stress state is realized.

In the case of plane stressed state, the tensor of stresses σ_{ij} ($i, j = 1, 2, 3$) in the polar system of coordinates is determined by the following components: $\sigma_r, \sigma_\varphi, \sigma_{r\varphi}$, tensor of deformations ε_{ij} ($i = 1, 2, 3$) by the components $\varepsilon_r, \varepsilon_\varphi, \varepsilon_{r\varphi}, \varepsilon_z$, deformation vector u by the components u_r, u_φ [1], the condition along the bounds of the wedge, i.e. for $\varphi = \frac{\pi}{2} - \alpha$ and $\varphi = \frac{\pi}{2} + \alpha$ is satisfied if for stress components we accept:

$$\sigma_r = -\frac{kP \cos \varphi}{rh(t)}, \quad (1)$$

$$\sigma_\varphi = 0, \quad \sigma_{r\varphi} = 0. \quad (2)$$

We determine the constant multiplier k from the equilibrium equation:

$$\int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}+\alpha} \sigma_r r \cos \varphi d\varphi = -P/h(t) \quad (3)$$

Allowing for formula (1), from (3) we find the constant k :

$$k = \frac{2}{2\alpha - \sin 2\alpha}$$

Thus, the radial stress σ_r is determined by the formula

$$\sigma_r = -\frac{2P}{2\alpha - \sin 2\alpha} \cdot \frac{\cos \varphi}{rh(t)}, \quad \left(\frac{\pi}{2} - \alpha \leq \varphi \leq \frac{\pi}{2} + \alpha \right). \quad (4)$$

On the bound $\varphi = \frac{\pi}{2} + \alpha$ we have:

$$\sigma_r = \frac{2P}{2\alpha - \sin 2\alpha} \cdot \frac{\sin \alpha}{rh(t)}.$$

Note that the solution of (2)-(4) will be exact for the points distant from the vertex of the wedge by $r = b \neq 0$.

The strain components are defined by the formulas [1]:

$$\varepsilon_r = \sigma_r / E, \quad \varepsilon_\varphi = -\nu \sigma_r / E, \quad \varepsilon_{r\varphi} = 0, \quad \varepsilon_z = -\nu \sigma_r / E. \quad (5)$$

The displacement vector components will be:

$$u_r = \varepsilon_\varphi r = -\nu \sigma_r / E, \quad u_\varphi = 0. \quad (6)$$

In formulas (5) and (6), the Poisson and Young ratio are denoted by ν and E .

The quantity σ_r that enters into formulas (5) and (6) is determined by formula (4). Therefore, the quantity $h(t)$ that enters into these formulas is unknown and should be defined. By the problem condition it should be found as a results of solution of corrosive wear of the plate's material. We will use the simplest model of wear [2]:

$$\frac{dh}{dt} = -\nu, \quad (7)$$

where ν is the corrosion rate determined from corresponding experiments for each investigated "metal-aggressive medium" system. The quanti-

ty ν is usually assumed to be a constant quantity for each point of the system.

The initial condition for equation (7) is the following: $h|_{t=0} = h_0$, where h_0 is the initial value of thickness. Therewith, from (7) we have:

$$h(t) = h_0 - \nu t. \quad (8)$$

Equation (7) makes sense for $t \leq t_s$, where t_s is the time of total corrosive wear of the wedge and is determined from the relation $h(t_s) = 0$. From (8) we have: $t_s = h_0 / \nu$. The time t_s is the greatest among all possible values of serviceability time of the wedge.

Now suppose that at time $t_b < t_s$ the wedge fails by stress. The given phenomenon may hold also in the case if the wedge's stress achieves the ultimate strength σ_b . We write this condition in the form: $\sigma_+ = \sigma_b$, where σ_+ is stress intensity that in conformity to our problem has the following form: $\sigma_+ = |\sigma_r|$. Therewith, from the equality $|\sigma_r| = \sigma_b$ allowing for (4) we determine:

$$h(t_b) = \frac{2P}{\sigma_b(2\alpha - \sin 2\alpha)} \frac{\cos \varphi}{r}. \quad (9)$$

Formula (9) determines the thickness of the considered wedge-shaped plate at corrosive failure time. By using (8), from (9) we find the time to corrosive failure of the wedge:

$$t_b = \frac{1}{\nu} \left[h_0 - \frac{2P}{\sigma_b(2\alpha - \sin 2\alpha)} \frac{\cos \varphi}{r} \right]$$

The failure of the point $r = b$, $\varphi = \frac{\pi}{2} + \alpha$ begins at time:

$$t_b = \frac{1}{\nu} \left[h_0 - \frac{2P}{\sigma_b(2\alpha - \sin 2\alpha)} \frac{\sin \alpha}{b} \right]. \quad (10)$$

Now we determine the condition at which there exists failure time of the wedge. Using the condition of existence of failure time ($t_b > 0$), from (10) we define restriction on the loading value:

$$P < \frac{h_0 b \sigma_b (2\alpha - \sin 2\alpha)}{2 \sin \alpha}. \quad (11)$$

Consequently, to prevent the instant failure of the wedge, the concentrated force applied to it should satisfy the condition (11).

Conclusions

Nonstationary stress-strain state of a corroded plate in the form of a wedge under the action of concentrated force applied to its vertex in the direction perpendicular to the symmetry axis, is determined. Time to corrosive failure of this plate is defined. Restriction on the value of the acting force, that enables to prevent instant failure of the wedge, is found.

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**KORROZIYAYA UĞRAYAN PAZIN SİMMETRİYA OXUNA PERPENDİKULAR İSTİQAMƏTDƏ
YÖNƏLƏN TOPA QÜVVƏ TƏSİRİNDƏN GƏRGİNLİK VƏZİYYƏTİ
VƏ UZUNÖMÜRLÜLÜYÜ**

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Elastiki deformasiyalanan, izotrop, korroziyaya uğrayan paza topa qüvvənin təsiri haqqında məsələ həll olunur. Topa qüvvə pazın tərəp nöqtəsində simmetriya oxuna perpendikulyar istiqamətdə təsir edir. Məsələnin həlli nəticəsində pazın korroziya yeyilməsində mövcud ola biləcəyi zaman təyin edilmişdir. Eyni zamanda pazın böhran vaxtının varlığı şərtindən topa qüvvəyə qoyulan məhdudluq şərti tapılmışdır.

Açar sözlər: paz, topa qüvvə, korroziya yeyilməsi, uzunömürlülük

**НАПРЯЖЕННОЕ СОСТОЯНИЕ И ДОЛГОВЕЧНОСТЬ КОРРОДИРУЮЩЕГО КЛИНА С
СОСРЕДОТОЧЕННОЙ СИЛОЙ, ПРИЛОЖЕННОЙ В ПЕРПЕНДИКУЛЯРНОМ
К ОСИ СИММЕТРИИ НАПРАВЛЕНИИ**

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Статья посвящена актуальной проблеме определения напряженного состояния и срока несущей способности пластины в форме клина с учетом действия окружающей среды. Сформулирована постановка математической задачи, где к уравнениям теории упругости добавляются уравнения коррозионного процесса. Решена поставленная задача, в результате чего найден срок несущей способности рассматриваемого клина в коррозионном процессе.

Ключевые слова: клин, сосредоточенная сила, коррозионный износ, долговечность