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THE SOLUTION OF THE CAUCHY PROBLEM FOR A LANGMUIR CHAIN WITH AN ASYMPTOTICALLY PERIODIC INITIAL CONDITION

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The Cauchy problem for a Langmuir chain with an asymptotically periodic initial condition is considered. The global solvability is proved. Formulas that allow to find a solution are obtained by the method of inverse scattering.

Keywords: Langmuir chain, Cauchy problem, scattering problem, inverse scattering method

Introduction

Langmuir chain, describing (see [1] and references therein) the induced scattering of plasma oscillations on the ions, has been an active subject of study for a number of years (see [1] - [5]) in the context of the theory of integrability. It is known that the inverse scattering method allows detailed investigation of the Cauchy problem for a Langmuir chain in the periodic case (see [4]). From a physical point of view it is natural and asymptotically periodic initial condition (see [5]). In this case, there arises the question of the existence of a solution of a Langmuir chain differing from a periodic one little.

For a sequence of positive functions $c_n = c_n(t) > 0$, $c_n \in C^{(1)}[0, \infty)$ consider the Cauchy problem for a Langmuir chain

$$\dot{c}_n = c_n(c_{n-1} - c_{n+1}), \quad n \in \mathbb{Z}, \quad (1)$$

$$c_n(0) = \hat{c}_n + x_n^0, \quad (2)$$

where $\sqrt{\hat{c}_{n+N}} = \sqrt{\hat{c}_n} > 0$ ($n \in \mathbb{Z}$, N is integer) is a periodic coefficient of Hill difference equation

$$\sqrt{\hat{c}_{n-1}}y_{n-1} + \sqrt{\hat{c}_n}y_{n+1} = \lambda y_n, \quad n \in \mathbb{Z} \quad (3)$$

and x_n^0 is an infinitesimal sequence.

Denote the periodic solution Langmuir

chain with the initial condition $\hat{c}_n(0) = \hat{c}_n$ as $\hat{c}_n(t)$. In [4], the existence of solutions $\hat{c}_n(t)$ is proved and a diagram of its construction is given. Assuming that $\hat{c}_n(t)$ is known, we seek such a solution $c_n(t) = \hat{c}_n(t) + x_n(t)$ of the problem (1)-(2) that the sequence $x_n(t)$ is a rapidly decreasing function, i.e. for any $T > 0$ satisfies the condition

$$\|S_1(t)\|_{C[0,T]} < \infty, \quad (4)$$

where $S_1(t) = \sum_{n \in \mathbb{Z}} |n| |x_n(t)|$.

In this paper, the solution of (1)-(2) is studied in the class (4) by the method of the inverse scattering problem. A similar problem was considered in [6], [7] for the Toda chain in [8] for the Korteweg-de Vries equation.

Theorem 1. *The solution of the problem (1) - (2) exists and is unique in the class (4) if $S_1(0) < \infty$.*

The theory of scattering for a perturbed Hill difference equation is used to solve problem (1)-(2) built in [7], [9]

$$\sqrt{\hat{c}_{n-1} + x_{n-1}^0}y_{n-1} + \sqrt{\hat{c}_n + x_n^0}y_{n+1} = \lambda y_n, \quad n \in \mathbb{Z} \quad (5)$$

where it is assumed that $S_1(0) < \infty$. It is known [9] that the continuous spectrum of the equation (5) consists of N zones $\sigma_n = [\lambda_{2n-1}, \lambda_{2n}]$, $n = 1, \dots, N$,

separated by gaps $\omega_0 = (-\infty, \lambda_1), \omega_n = (\lambda_{2n}, \lambda_{2n+1}), n = 1, \dots, N-1, \omega_N = (\lambda_{2N}, +\infty)$.

This equation may also have simple eigenvalues $\gamma_k, k = 1, \dots, p$ lying in the gaps.

Let Γ be complex λ -plane with cuts along the spectrum σ . As follows from [9], equation (5) has a solution $g_n^\pm(\lambda)$ that can be represented in the form

$$g_n^\pm(\lambda) = \alpha^\pm(n) \left\{ \psi_n^\pm(\lambda) + \sum_{m=n\pm 1}^{+\infty} K^\pm(n, m) \psi_m^\pm(\lambda) \right\}, n \in \mathbb{Z}, \quad (6)$$

where $\psi_n^\pm(\lambda)$ is a Floquet solution of the unperturbed equation (3). These decisions relate to the range of equalities

$$g_n^-(\lambda) = a(\lambda) \bar{g}_n^+(\lambda) + b(\lambda) g_n^+(\lambda), \lambda \in \partial\Gamma, \quad (7)$$

$$g_n^-(\gamma_k) = M_k g_n^+(\lambda), k = 1, \dots, p. \quad (8)$$

This set of values $\{a(\lambda), b(\lambda); \gamma_k; M_k; k = 1, \dots, p\}$ is called the scattering data for the equation (5).

We introduce the solutions $\varphi_n(\lambda), \theta_n(\lambda)$ of the equation (3) with the conditions $\varphi_0(\lambda) = \theta_1(\lambda) = 0, \varphi_1(\lambda) = \theta_0(\lambda) = 1$.

1. Let $\Delta(\lambda) = 2^{-1}[\varphi_{N+1}(\lambda) + \theta_{N1}(\lambda)]$. Let

$$m^\pm(\lambda) = \frac{\varphi_{N+1}(\lambda) - \theta_N(\lambda)}{2\varphi_N(\lambda)} \pm \frac{\sqrt{\Delta^2(\lambda) - 1}}{\varphi_N(\lambda)} \quad \text{be}$$

Weyl function where the regular branch of the radical is selected in Γ so that $\sqrt{\Delta^2(\lambda) - 1} < 0$ when $\lambda > \lambda_{2N}$.

It follows from the results of [7], [9] that, subject to condition $S_1(0) < \infty$ the disturbance factor x_n^0 is uniquely determined by the scattering data and on the periodic coefficients $\hat{c}_n$. That is, we suggest that

$$F(n, m) = - \sum_{k=1}^p \frac{\varphi_N(\gamma_k)}{2\sqrt{\hat{c}_N} \sqrt{\Delta^2(\gamma_k) - 1}} \frac{M_k}{a'_k(\gamma_k)} \psi_n^\pm(\gamma_k) \psi_m^\pm(\gamma_k) + \frac{1}{2\pi i} \int_{\partial\Gamma} \frac{\varphi_N(\lambda)}{2\sqrt{\hat{c}_N} \sqrt{\Delta^2(\lambda) - 1}} \frac{b(\lambda)}{a(\lambda)} \psi_n^+(\lambda) \psi_m^+(\lambda) d\lambda. \quad (9)$$

Then the coefficient x_n^0 is reduced by the formula

$$\frac{\hat{c}_n + x_n^0}{\sqrt{\hat{c}_n}} = \sqrt{\hat{c}_n} + \sqrt{\hat{c}_{n+1}} K^+(n, n+2) - \sqrt{\hat{c}_{n-1}} K^+(n-1, n+1), n \in \mathbb{Z} \quad (10)$$

wherein the core $K^+(n, m)$ is a solution of the basic equation of Gelfand-Levitan-Marchenko type

$$F(n, m) + K^+(n, m) + \sum_{r=n+1}^{+\infty} K^+(n, r) F(r, m) = 0, \quad n \in \mathbb{Z}, m \geq n+1. \quad (11)$$

In the case where the coefficient $\sqrt{\hat{c}_n(t) + x_n(t)}$ is in the equation (5) where $\sqrt{\hat{c}_n(t) + x_n(t)}$ is a solution of equation (1), the evolution of the scattering data is described by at the time by the following.

Theorem 2. *If in equation (5) with a factor $\sqrt{\hat{c}_n(t) + x_n(t)}$ which $c_n(t) = \hat{c}_n(t) + x_n(t)$ is a solution of equation (1), the evolution of the scattering data is described by formulas*

$$a(\lambda, t) = a(\lambda, 0), b(\lambda, t) = b(\lambda, 0) \exp \left(\int_0^t [\delta^-(\lambda, \tau) - \delta^+(\lambda, \tau)] d\tau \right), \quad (12)$$

$$M_k(t) = M_k(0) \exp \left(\int_0^t [\delta^-(\gamma_k, \tau) - \delta^+(\gamma_k, \tau)] d\tau \right),$$

$$k = 1, \dots, p,$$

where

$$\delta^\pm(\lambda, t) = \lambda m^\pm(\lambda, t) \sqrt{\hat{c}_0(t)} + \frac{\hat{c}_{-1}(t) - \hat{c}_0(t) - \lambda^2}{2}$$

and $m^\pm(\lambda, t) = \psi_1^\pm(\lambda, t)$ is Weyl function for the equation (3) with a factor $\sqrt{\hat{c}_n(t)}$.

Formulas (12) allow us to find the solution of problem (1)-(2). To do this it is necessary to find the data scattering when $t = 0$, then to build a function $F^+(n, m, t)$ by formula (9)

where it should substitute (12) instead of $a(\lambda, 0), b(\lambda, 0), M_k(0)$. Further the following equation of the type (11) with the parameter t should be solved. Then formula (10) recorded at the time t give a solution.

Note that if to suggest $N=1$, then the equation (12) becomes known evolutionary formulas for the case of rapidly decreasing initial data (see [1], [4]). In addition, if the periodic solution $\hat{c}_n(t)$ is built (at $N=2$ the solution $\hat{c}_n(t)$ is found by the formula $\hat{c}_n(t) = \hat{c}_n(0)$), then, as in the case of rapidly decreasing initial conditions, a soliton solution of the problem (1) - (2) can be found, taking the initial condition corresponding to the scattering data of $\{a(\lambda, 0), b(\lambda, 0) \equiv 0; \gamma_k; M_k(0)\}$ type.

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ASİMPTOTİK PERİODİK BAŞLANGIÇ ŞƏRTƏ MALİK OLAN LENGMÜR ZƏNCİRİ ÜÇÜN KOŞI MƏSƏLƏSİNİN HƏLLİ

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Asimptotik periodik başlangıç şərtə malik olan Lengmür zənciri üçün Koşi məsələsinə baxılır. Məsələnin global həlli isbat olunmuşdur. Tərs səpilmə metodu ilə həlli tapmağa imkan verən düsturlar alınmışdır.

Açar sözlər: Lengmür zənciri, Koşi məsələsi, səpilmə məsələsi, tərs səpilmə metodu

РЕШЕНИЕ ЗАДАЧИ КОШИ ДЛЯ ЛЕНГМЮРОВСКОЙ ЦЕПОЧКИ С АСИМПТОТИЧЕСКИ ПЕРИОДИЧЕСКИМ НАЧАЛЬНЫМ УСЛОВИЕМ

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Рассмотрена задача Коши для ленгмюровской цепочки с асимптотически периодическим начальным условием. Доказана глобальная разрешимость задачи. Методом обратной задачи рассеяния получены формулы, позволяющие найти решение.

Ключевые слова: ленгмюровская цепочка, задача Коши, задача рассеяния, метод обратной задачи рассеяния