

BASICS OF MATHEMATICAL MODELING OF GAS LIFT PROCESS IN THE WELL-RESERVOIR SYSTEM

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The paper is devoted to the problem of mathematical modeling of the gas-lift wells in the well-reservoir system. Results of field experiments carried out in the well of the offshore field "Neft dash-lari" (Azerbaijan) are represented. Has been formulated the basic principles of modeling of the gas-lift wells. An approach is proposed to account for the real processes at the zone of the gas-liquid mixture for modeling the considered process. This takes into account the dynamic processes occurring in the wellbore and in the reservoir, real properties of fluids and porous medium during filtration.

Keywords: well-reservoir system, gas-lift, optimal control, mathematical modeling, simulation, well testing

Introduction

Gas lift is one of the most widely applied methods of oil well operation [4, 6, 7]. Despite the wide interest shown by researchers to the problems of mathematical modeling and optimal control of gas lift, they are still not fully resolved. The obtained solutions to this day, such as [2, 7, 9] are of limited interest for practical applications, while they have created an important theoretical basis.

So in [1] has been noted the need to consider the process in the well-reservoir system. In this paper a system of differential equations describing the motion of the injected gas and liquid mixture (GLM) in the well has been obtained. It describes the motion of the GLM in the well. The offered system of differential equations allows us to determine the pressure distribution and flow rate along the well pipes and has the following form:

$$-\frac{\partial p}{\partial x} = \frac{1}{F} \frac{\partial Q}{\partial t} + \frac{2a}{F} Q, \quad 2a = \frac{g}{w_c} + \frac{\lambda_c w_c}{2D} \quad (1)$$

$$-\frac{\partial p}{\partial t} = \frac{c^2}{F} \frac{\partial Q}{\partial x}, \quad (2)$$

where $Q = F\rho w_c$ - gas mass flow rate in a given section of the annulus (or gas-liquid mixture in the lift), F - cross sectional area the annulus (for case of flow of the injected gas) and the

lifting pipes (in the case of flow of the GLM), w_c - average speed of the flow (gas or GLM), ρ - medium density, c - the speed of sound in the medium.

On the basis of the motion equations (1) and (2) carried out a number of studies in the area of optimal control of the gas-lift wells by various authors [2]. For example, within the representations formulated in [1], the well-reservoir system is considered consisting of the following four zones: injection zone (in this case the annulus), which varies in the range $0 \leq x \leq (L-0)$, "the area" $(L-0, L+0)$, where the injected gas is mixed the reservoir fluid and forms a GLM, the lifting zone $(L+0) \leq x \leq 2L$ (lift pipes) and the reservoir, which shows itself in the overall dynamics of the system by the hydrodynamics of the reservoir fluids influx to the well bottom. It is assumed that the production rate is the sum of the injected gas and reservoir fluids: $Q_{GLM} = Q_{inj} + Q_o$.

By authors of aforementioned papers has been formulated and solved a number of control problems for gas lift under conditions of constant of Q_o . The simplified conditions taken in the marked and similar investigations led to the qualitatively different curve of $Q_{GLM}(Q_{inj})$ from the characteristic curves known from laboratory experiments [4, 6] and well testing performed by us.

This confirms that taking into account the zone of formation of GLM, i.e. the mathematical description of the process of formation of GLM is crucial in the simulation of the gas lift process.

The purpose of this paper is to formulate the basic principles of mathematical modeling of gas-lift taking into account features of formation of GLM and dynamic processes that occur in the well-reservoir system. Within this study, we neglect the volume of the GLM formation zone and assume that the injected gas is mixed with the fluid at a point $x = L$, where L is the length of the lifting tube. Considering the marked assumptions we shall assume that the injected gas into the annulus moves in $0 \leq x \leq (L-0)$ and the GLM flow occurs within the $(L+0) \leq x \leq 2L$.

The results of the well production testing

To study the features of the process in the actual gas lift well we performed numerous well experiments. For this purpose has been chosen the well number of 2390 in the field "Neft Dashlari", which operates an oil reservoir located in the depth of 620 meters. The stage of static level of the oil in the well bore was 500 m. Were considered different variants, both in the immersion depth (setting depth), and the diameter of the lift in order to study of the influence of immersion depth and diameter of the lifting pipes. Part of the results of experiments is illustrated in figure 1, where we consider two variants of lift diameter, which were equal to 38.1 and 50.8 mm. In addition, to investigate the influence of the immersion depth when the diameter is 38.1 mm the lift experiment was carried out at three different values of lift's lengths - 260, 345 and 410 m, which is equivalent to the immersion depth of 190, 275 and 340 meters, respectively.

In the case when the diameter is 63.5 mm experiments were carried out with lifts length of 528 and 547 meters, i.e. at immersion depth of 458 and 477 meters, respectively.

Looking at the results shown in figure 1 it can easily be seen that with increasing the diameter of the pipes in order to achieve the maximum point of the dependence $Q_o(Q_{inj})$ it is required more flow rate of injected gas, and at

the same diameter of pipes the increasing of the immersion depth of lift leads to an increase the numerical value of the well production rate at this point.

This is confirmed by the results of laboratory experiments carried out in short lifts [4].

From the results of laboratory experiments [4] is also known that under some circumstances, steepness of the Q_o versus Q_{inj} curve is changed. This can happen for example by increasing the water influx of the well at a relatively high well production rate, which leads to a change in PVT properties of the liquid phase. Thus, with decreasing density and viscosity of the reservoir fluid the steepness of the characteristic curve $Q_o(Q_{inj})$ decreases. This can be seen from the curves of variants $l=547$ and $l=528$ at $d=50.8\text{ mm}$ in figure 1.

Taking into account these factors that affect the considered process, the dependence of the well production rate of gas injection rate $Q_o(Q_{inj})$ should be described by an expression of following type: $Q_o = \delta_3 [Q_{inj} - \delta_2]^2 + \delta_1$, (3) where δ_1 , δ_2 , δ_3 - the proportionality factors characterizing the internal diameter of the lift, the relative immersion depth and the rheological properties of reservoir fluids.

Taking into account this, below we will consider the problem of modeling the process of gas lift in the following formulation.

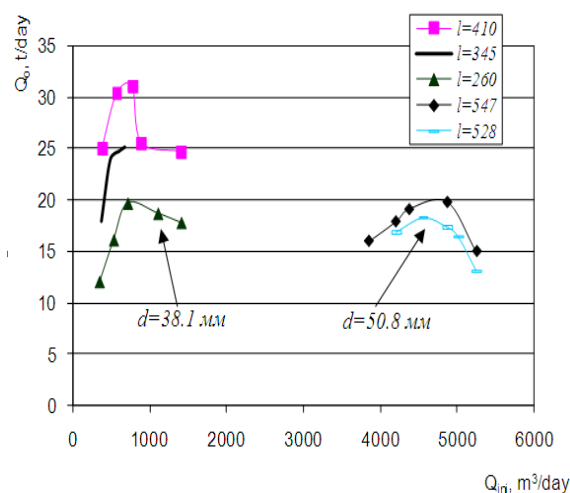


Fig 1. Characteristic curves for different diameters (d) and lengths (l) of the lift

Simulation of the GLM formation zone

To describe the motion of the injected gas in the annulus and movement of GLM in the lift we shall use equations (1) and (2). Applying the averaging method they can be rewritten as the following ordinary differential equations [1]:

$$\frac{dQ_i}{dx} = \frac{2a_i w_{c_i}}{w_{c_i}^2 - c_i^2} Q_i, \quad 2a = \frac{g}{w_c} + \frac{\lambda_c w_c}{2D} \quad (4)$$

$$\frac{dp_i}{dx} = \frac{2a_i c_i^2}{F_i (w_{c_i}^2 - c_i^2)} Q_i, \quad (5)$$

where $i = 0$ or 1 . In the case when $i = 0$, the equations express the motion of the gas in the annulus within the range $0 \leq x \leq (L - 0)$ in the following boundary conditions:

$$\begin{aligned} p(0) &= p_0, \\ Q(0) &= Q_0. \end{aligned} \quad (6)$$

We use the following nomenclature:

$$\begin{aligned} p(L - 0) &= p_L, \\ Q(L - 0) &= Q_L. \end{aligned}$$

And in the lift, i.e when $i = 1$, taking into account the above considerations, and the expression (3) we use the following boundary conditions:

$$\begin{aligned} p(L + 0) &= p_z = p_L + p_d, \\ Q(L + 0) &= Q(L - 0) + \delta_3 [Q(L - 0) - \delta_2]^2 + \delta_1, \end{aligned} \quad (7)$$

where p_z is the actual bottomhole pressure; p_d is velocity pressure (dynamic pressure) which is created by the dynamic liquid column in the annulus; The coefficients δ_1 , δ_2 and δ_3 in the case of a certain diameter and immersion depth of the lift can be determined with the well production data. So for three regime of gas injection we can write following expressions:

$$\begin{aligned} Q_j^1(2L) &= \delta_3 [Q_0^1(L + 0) - \delta_2]^2 + \delta_1, \\ Q_j^2(2L) &= \delta_3 [Q_0^2(L + 0) - \delta_2]^2 + \delta_1, \\ Q_j^3(2L) &= \delta_3 [Q_0^3(L + 0) - \delta_2]^2 + \delta_1, \end{aligned} \quad (8)$$

where $Q_j^1(2L)$, $Q_j^2(2L)$, $Q_j^3(2L)$ are well production rates at the injected gas flow rates of Q_0^1 , Q_0^2 and Q_0^3 , respectively.

When well production data are available at various gas injection regimes, with some assumption by (8) we can determine the coefficients δ_1 , δ_2 , δ_3 .

It is should be noted that, to improve the reliability of determining the coefficients δ_1 , δ_2 , δ_3 it is necessary to increase the number of well test data. In this case for processing the well production test data to determine the coefficients can be use the method proposed in [3].

So the system of equations (4) and (5) with the boundary conditions (6), (7) and taking into account the calculated values of the coefficients δ_1 , δ_2 , δ_3 allows us to solve various problems of gas lift, including the problems of prediction and control.

To test the approach outlined above were carried out a series of numerical calculations, which were based on the following initial data:

The length of the lift $L = 620$ m; The effective diameter and the cross-sectional area of the annulus $D_0 = 0.066$ m and $F_0 = 0.003402$ m²; The inner diameter of the lift $D_1 = 0.038$ m.

In the calculations, the values of the speed of sound c at annulus and the lift were taken equal to 430 and 1500 m/s, respectively. And coefficients of total drag in the annulus and the lift was taken to be 0.0027 and 0.01, respectively. Here we have used the production test data of the well №2390 of the field "Neft dashlari" (Azerbaijan) at the various gas injection regimes. They are shown in tables 1 and 2.

Table 1

Gas injection conditions

Variants	$Q_0, m^3/s$	$w_0, m/s$	p_0, MPa
1.	0.0040	1.1758	1.7
2.	0.0062	1.8225	1.9
3.	0.0083	2.4397	2.3
4.	0.0128	3.7625	2.6
5.	0.0163	4.7913	2.8

Table 2

The dependence of the well production rate (well number of 2390) on the gas flow rate when the lift's diameter $d = 38 \text{ mm}$ and immersion depth $l = 190 \text{ m}$ (the length of the lift is 260 m)

$Q_0, \text{ m}^3/\text{s}$	$Q_j(2L), \text{ m}^3/\text{s}$
0.0040	0.0001850
0.0062	0.0002483
0.0083	0.0003039
0.0128	0.0002885
0.0163	0.0002746

To this end, at first using expressions (8) taking into account the data of table 2 were calculated values of the coefficients δ_1 , δ_2 , δ_3 . They had the following values:

$$\delta_1 = 3.149052 \cdot 10^{-4}, \delta_2 = 1.006541 \cdot 10^{-2}, \delta_3 = -3.531066.$$

Then, using the conditions (6) and (7) considering the data of table 1 by equations (4) and (5) were calculated values of the functions $p_0(x)$ and $Q_0(x)$ for various values of the injected gas flow rate. For this purpose, we used the method of Euler. In the calculations, the bottomhole flowing pressure was assumed to be constant and equal to the 15.0 MPa. In addition, in the first approximation it is assumed that the expressions (8) are also true at the boundary $x = L$. Given this assumption, the flow rate of formation fluids at the boundary $x = L$ was calculated by the following expression:

$$Q_j(L+0) = \delta_3 [Q_0(L-0) - \delta_2]^2 + \delta_1. \quad (8)$$

The results of these calculations are illustrated in figure 2 and figure 3.

Figure 2 shows the dependence of the well flow rate on the gas injection rate within the conditions considered. It is seen that the increase in the gas injection rate after reaching a certain value leads to a decrease in oil production, which fully coincides with the practice [4]. In this case, the maximum oil production rate corresponds to the gas flow rate of $900 \text{ m}^3/\text{day}$.

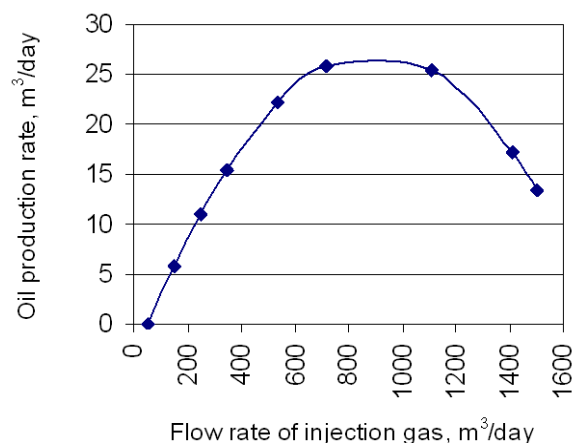


Fig. 2. Dependence on oil production rate on the gas injection rate

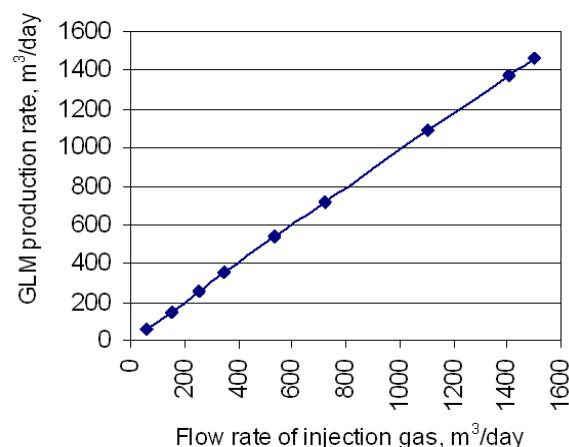


Fig. 3. Dependence of GLM production rate on the gas injection rate

Despite this fact, well production by gas-liquid mixture increases with increasing of the gas injection rate almost linearly (figure 3). Comparing the data of figure 2 and figure 3, it is not difficult to see that this is due with the fact that the volume of gas is much greater than the volume of oil in the mixture. Therefore, a further decrease of the oil in the mixture is not visible in the considered curve.

Application of the above stated approach in practice requires the definition of the dynamics of the liquid level in the well and consequently, the bottomhole pressure. So, the next section of the paper is devoted to the problem of determining the dynamic pressure (i.e., the pressure created by the dynamic liquid column in the well) p_d included in the boundary condition (7).

Determination of the dynamic fluid level and the bottomhole pressure

It is obvious that dynamics of the process of gas lift in a well-reservoir system is created by the pressure of the injected gas, the dynamics of oil production rate and influx from the reservoir under the considered bottomhole pressure. In other words, the entire process has three main components: the excess pressure (overpressurization) of the injected gas, the oil production rate of the well and the third, which depends on these two factors, the influx of formation fluids to the bottomhole.

The presented approach allows determining the first two components. Now consider the problem of determining the bottomhole pressure at any time. Assume that the well production rate and the influx rate of reservoir fluids are $q_d = Q_j(2L)$ and $q_o(t)$, respectively, where according to (8) $Q_j(2L) = \delta_3 [Q_1^1(L+0) - \delta_2]^2 + \delta_1$.

It is known that the static fluid level in the well creates a pressure to the bottom hole nearly equal to the reservoir pressure, thereby preventing the formation fluid influx to the bottom hole. taking away any volume of fluid from the wellbore creates a certain depression and leads to a resumption of formation fluid influx to the bottom hole. This leads to an increase in fluid level that turn leads to an increase in bottomhole pressure and finally of termination of the influx.

The time t_d during which the liquid column up to the level of h_d , divide into sufficiently small segments equal Δt . Taking into account that the volume of the liquid column to this moment is equal to $V_d = \int_0^{t_d} [q_o(t) - q_d(t)] \cdot dt$, then h_d can be defined as the sum of Δt on condition $V_d \approx \sum_{i=0}^n [q_{oi}(t) - q_d(t)] \Delta t$. Given that $V_d = \frac{\pi r_w^2 p_z}{\gamma_o g}$ or $V_d = \pi r_w^2 h_d$, then to determine the height of the liquid column and the corresponding pressure we write the following expression:

$$h_d = \frac{\sum_{i=0}^n [q_{oi}(t) - q_{di}(t)] \Delta t}{\pi r_w^2} \text{ or}$$

$$p_z = \frac{\sum_{i=0}^n [q_{oi}(t) - q_{di}(t)] \Delta t \cdot \gamma_o g}{\pi r_w^2},$$

where r_w , γ_o , g are the radius of the well, the oil density and gravitational acceleration.

On the basis of expression (9) developed an algorithm that allowed to create a computer model to determine the temporal characteristics of the considered process. Using this program were carried out some computer experiments. They were based on the following initial data:

Well radius $r_w = 0.1$ m; radius of the well-drainage area $R_b = 1000$ m; reservoir pressure $p_b = 5.0$ MPa, formation thickness $h_f = 50$ m; absolute rock permeability $k_0 = 0.1 \cdot 10^{-12}$ m².

The results are presented in figure 4. Graphs of dependences $q_o(t)$ and $h(t)$ in figure 4 illustrate the nature of reducing the influx of oil q_o and raising the level of fluid h_d in the well over time. It is seen that the growth of the oil column in the well is accompanied by the intense influx drop. Then we see the decline in tempo of considered process. So, is reduced the rates of influx and the oil level rise. As a result, after 12000 seconds the influx is reduced so much, at which the oil level rise rate becomes insignificant. And further, this process continues for a long time.

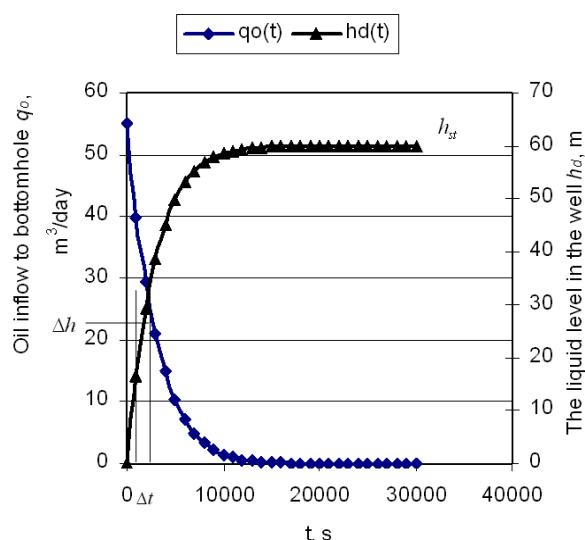


Fig. 4. The dynamics of influx (q_o) and the liquid level (h_d) in the well

In computer calculations to determine the Influx to the wellbore q_o used a special algorithm, which is discussed in the next section.

Influx to the wellbore

Having data on the dynamics of change of the bottomhole pressure the influx can be determined for any time. For this purpose, can be used the algorithm described in [8], which takes into account the real properties of the reservoir system. According to the marked article, the influx to the well can be described by the following expression:

$$q_o = \frac{2\pi h_r (H_b - H_c)}{\ln \frac{R_b}{r_w} - \frac{1}{2}}, \quad (10)$$

where H is a function similar Christianovich's function, H_b, H_w are values of the H functions on the well-drainage area boundary and at the bottomhole, respectively. It should be noted that H_w is a function of the p_z , which is defined by (9).

So having the value of the influx at each moment in time can be defined a liquid level in the well using the algorithm described above.

Determination of the current reservoir parameters

Further, to determine the main parameters of reservoir systems, such as average reservoir pressure and oil saturation has been obtained a system of differential equations on base the material balance equation of oil and gas. In this case, the hydrocarbon system is presented as consisting of two components (pseudo components) and the two phases. In addition, is taken into account relaxation and the elastic deformation of the reservoir [8]:

$$\frac{dp}{dt} = - \frac{\frac{q_o}{\Omega_o \Omega} (G\alpha_4 + \alpha_2) - (\alpha_2\alpha_3 + \alpha_1\alpha_4) \frac{1}{\Omega} \frac{\bar{m}(p) - \bar{m}}{\tau_m}}{(\alpha_5 + \alpha_6)\alpha_4 + (\alpha_7 + \alpha_8)\alpha_2}, \quad (11)$$

$$\frac{ds}{dt} = - \frac{\frac{q_o}{\Omega_o \Omega} + (\alpha_7 + \alpha_8) \frac{dp}{dt} + \alpha_3 \frac{1}{\Omega} \frac{d\bar{\Omega}}{dt}}{\alpha_4}, \quad (12)$$

$$\frac{d\bar{\Omega}}{dt} = a_m \exp[a_m(p - p_0)] \frac{dp}{dt}, \quad (13)$$

where $\Omega(p, t) = \pi(R_b^2 - r_w^2)h_r m(p, t)$ is a pore volume of the drainage area of the well.

The system (11)-(13) is solved by one of the approximate methods.

The calculation algorithm

The algorithm for the calculation of the main parameters of the gas lift can be represented in a simplified form as follows:

1. Are defined the excess pressure and the injected gas flow rate at bottomhole (i.e. when $x = (L - 0)$) using (4) and (5) considering the boundary conditions (6);

2. Given the values of the excess pressure of the injected gas, calculated in the previous step is determined the fluid influx to the wellbore by the expression (10) using the procedure described in [8];

3. Is determined the current liquid level into the wellbore and the bottomhole pressure by the expressions (9);

4. Is determined the flow rate of gas-liquid mixture in the entry of the lift pipes by the expression (8') with known values of $\delta_1, \delta_2, \delta_3$;

5. Using the results obtained in the third and fourth stages, as the boundary conditions is calculated well production rate and wellhead pressure by the solution of equations (4) and (5) in the case of $i=1$;

6. Are determined the average reservoir parameters by solving the system of equations (11) - (13) and is repeated steps 1-5.

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QUYU-LAY SİSTEMİNDƏ QAZLIFT PROSESİNİN RİYAZİ MODELƏŞDİRİLMƏSİNİN ƏSAS PRİNSİPLƏRİ

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Məqalə qazlift prosesinin quyu-lay sisteminə baxılan halda riyazi modelləşdirilməsi probleminə həsr olunub. “Neft daşları” yatağında aparılmış mədən eksperimentlərinin nəticələri verilir və bu nəticələr əsasında qazlift quyusunun riyazi modelləşdirilməsinin əsas prinsipləri formalaşdırılır. Baxılan prosesin modelləşdirilməsi zamanı qaz-maye qarışığının yaranması zonasında real proseslərinin nəzərə alınması üçün yanacaq təqdim olunur. Bu zaman quyuda və layda baş verən dinamik proseslər, layda süzülmə zamanı lay flüidlərinin və məsaməli mühitin real xüsusiyyətləri nəzərə alınır.

Açar sözlər: qazlift, modelləşdirmə, quyu-lay sistemi, məsaməli mühit, süzülmə, qaz-maye qarışığı, quyu tədqiqatı

ОСНОВНЫЕ ПРИНЦИПЫ МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ ПРОЦЕССА ГАЗЛИФТА В СИСТЕМЕ СКВАЖИНА-ПЛАСТ

Фикрет А. Алиев, М.А.Джамалбеков

Статья посвящена проблеме математического моделирования процесса газлифта в системе скважина-пласт. Приводятся результаты промысловых экспериментов, проведенных в месторождении “Нефть Дашлары”, на основе которых формируются общие принципы математического моделирования процесса газлифта. Предлагается подход моделирования рассматриваемого процесса, учитывающий реальные процессы, происходящие в зоне образования газожидкостной смеси. При этом учитываются динамические процессы, происходящие в скважине и пласте, и реальные свойства пластовых флюидов и пористой среды при фильтрации.

Ключевые слова: газлифт, моделирование, система скважина-пласт, пористая среда, фильтрация, газожидкостная смесь, исследование скважины