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Condensed Matter physics

TOPOLOGICAL SUPERCONDUCTIVITY IN QUASI-ONE DIMENSIONAL STRUCTURES

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A new class of superconducting quasi-one dimensional materials was suggested, where the topological phase is realized even in the absence of an external magnetic field. Such time-reversal invariant topological superconductors are fabricated by filling the superconducting materials into the periodic channel of dielectric matrices like zeolite and asbestos under high pressure. The topological superconducting phase sets up in the presence of large spin-orbit interactions when s-wave intra-wire and d-wave inter-wire pairings take place. Kramers pairs of Majorana bound states emerge at the edges of each wire.

Keywords: *superconductivity, Majorana fermions, topological dielectrics*

Introduction. The recent theoretical prediction and experimental observation of topological phase of matter in a number of materials [1] have attracted great interest in this subject. These qualitatively new materials are characterized by a bulk gap and topologically protected gapless edge states. The topological concept can be applied to both insulators and superconductors. The time-reversal invariant (TRI) topological superconductors (SCs) was predicted by theoretical classification of Bogolyubov-de Gennes (BdG) Hamiltonian [2,3] which constitute a completely distinct symmetry class DIII. Topological SC has a bulk gap and gapless Andreev bound states. Due to the presence of intrinsic particle-hole symmetry (PHS) the gapless zero-modes in the topological superconductors constitute Majorana fermions (MFs), obeying non-Abelian braiding statistics which is useful in implementing fault-tolerant topological quantum computer [4]. A variety of condensed-matter systems hosting localized Majorana quasi-particles have been proposed, notably quantum Hall states [5] and topological superconductors [6-9], as well as systems with the charge-density wave instability [10]. The challenge is to find a real material which supports the topological SC properties.

Recent works [11,12] have proposed TRI topological superconductivity (class DIII) with a Z_2 invariant, which takes a value $\nu = 1$ in a

number of systems with intrinsic or proximity induced superconductivity of p-wave, spin-triplet or d-wave and $s_{\pm}$ -wave spin-singlet pairings.

In this work we show that the time-reversal invariant topological superconducting phase can be realized in quasi-one-dimensional (quasi-1D) wires on the plane in the presence of s-wave intra-wire and d-wave inter-wires pairings. Similar structures have been fabricated [13] by filling a superconducting material into the cavities or channels of dielectric matrices like zeolite and asbestos crystals under high pressure up to 30 kbar. The regular set of channels or cavities of $5 - 10 \text{ \AA}$ diameter in zeolite and from $20 - 30 \text{ \AA}$ up to $100 - 150 \text{ \AA}$ diameter in asbestos [14] form a periodic lattice of different geometry in one-, two- and three-dimension, e.g. like several equidistant filaments with $5 \div 20 \text{ \AA}$ separation in zeolite and with $150 \div 500 \text{ \AA}$ separation in asbestos on a plane or quasi-1D space lattice. The critical temperatures T_c of such structures become higher than the T_c of the bulk superconductors by factors of $2 - 5$ [13]. High stress field around the filaments may guarantee higher value of spin-orbit interactions in the structures. An increase in the critical temperature may be caused by an additional inter-wire pairings due to polarization of the

dielectric matrix between the wires. Multiple Majorana-Kramers pairs with strongly spatial overlapping wave functions are protected by time-reversal symmetry, and they persist at zero energy. The picture is rather complicated in the quasi-1D topological superconductors, where the Majorana-Kramers pairs emerge at the ends of each wires. The Majorana-Kramers pairs with spacially overlapping wave functions can persist only if their splitting is forbidden by an underlying symmetry such as spatial reflection. In order to prove a protection of the Majorana fermions at each end of the superconducting wire, one can reduce N-superconducting wires problem to one wire problem, and to clarify a survival condition for the Majorana-Kramers pairs in this effective wire.

The paper is structured as follow: In Section II we describe the model of N weakly coupled superconducting wires on a plane with intra-wire and inter-wire pairings. In this Section we show that N-superconducting wires problem can be reduced to two-wires one. The energy spectrum of the TRI topological SC, which hosts a zero energy bound states in the absence of the Zeeman magnetic field, is calculated, and conditions for realization of the TRI topological superconducting phase are discussed in Section III.

Model for strongly anisotropic topological superconductors. The equidistant superconducting wires, aligned along x-axes in $\{x, y\}$ plane, with intra- and inter-wire s-wave pairings (see, Fig.a), in the presence of spin-orbit interactions (SOIs) and arbitrary directed magnetic field B are described by Hamiltonian

$$\begin{aligned} \hat{H} = \sum_j \{ \hat{H}_{j,j} + \hat{H}_{j,j+1} + \hat{H}_{j+1,j} \}, \quad (1) \\ \hat{H}_{j,j} = \sum_{\sigma, \sigma'} \int \frac{dk_x}{2\pi} \{ \psi_{j,\sigma}^+(k_x) \xi_{k_x} \psi_{j,\sigma}(k_x) + \\ + 2 \sin k_x \psi_{j,\sigma}^+ [\alpha(\sigma_y)_{\sigma, \sigma'} + \beta(\sigma_x)_{\sigma, \sigma'}] \psi_{j, \sigma'} + \\ + \varepsilon_z \psi_{j,\sigma}^+ [(\sigma_z)_{\sigma, \sigma'} \cos \theta + \\ + (\sigma_x)_{\sigma, \sigma'} \sin \theta \cos \varphi + \end{aligned}$$

$$+ (\sigma_y)_{\sigma, \sigma'} \sin \theta \sin \varphi] \psi_{j, \sigma'} + \\ + [\Delta_0 \psi_{j, \uparrow}^+(k_x) \psi_{j, \downarrow}^+(-k_x) + h.c.] \}, \quad (2)$$

$$\begin{aligned} \hat{H}_{j,j+1} = \sum_{\sigma, \sigma'} \int \frac{dk_x}{2\pi} \{ t_{\perp} [\psi_{j,\sigma}^+(k_x) \psi_{j+1,\sigma}(k_x) + h.c.] + \\ + \frac{i}{d} \bar{\alpha} [\psi_{j,\sigma}^+(\sigma_x)_{\sigma, \sigma'} \psi_{j+1,\sigma'} - \psi_{j+1,\sigma}^+(\sigma_x)_{\sigma, \sigma'} \psi_{j,\sigma'}] + \\ + \frac{i}{d} \bar{\beta} [\psi_{j,\sigma}^+(\sigma_y)_{\sigma, \sigma'} \psi_{j+1,\sigma'} - \psi_{j+1,\sigma}^+(\sigma_y)_{\sigma, \sigma'} \psi_{j,\sigma'}] + \\ + [\Delta_0 (\psi_{j,\uparrow}^+(k_x) \psi_{j+1,\downarrow}^+(-k_x) + \\ + \psi_{j+1,\uparrow}^+(k_x) \psi_{j,\downarrow}^+(-k_x) + h.c.] \}, \quad (3) \end{aligned}$$

where $\hat{H}_{j+1,j} = \hat{H}_{j,j+1}^+$, $\psi_{j,\sigma}^+(k_x) (\psi_{j,\sigma}(k_x))$ is a creation (annihilation) operator of an electron with a longitudinal momentum k_x and spin σ in a j th wire $t_{\parallel} (t_{\perp})$ and μ are the longitudinal (transverse) overlap integral and the Fermi energy, $\xi_{k_x} = (-2t_{\parallel} \cos k_x - \mu)$ is the energy

dispersion in a single wire, $\alpha(\beta)$ and $\bar{\alpha}(\bar{\beta})$ are the longitudinal and transverse components of Rashba (Dresselhaus) spin-orbit constants; $\varepsilon_z = g\hbar\mu_B B/2$ is the Zeeman energy θ and φ are correspondingly the polar and the azimuthal angles of the magnetic field $B = B\{\sin \phi \cos \phi, \sin \theta \sin \theta, \cos \theta\}$. The existence of intra- and inter-wire s-wave order parameters Δ_0 and Δ_1 allows one to introduce the effective order parameter $\Delta(k_y)$ in momentum space, $\Delta(k_y) = \Delta_0 + 2\Delta_1 \cos k_y = |\Delta(k_y)| e^{i\phi}$.

Further, we will take $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta}$. We assume also that the structure is strongly anisotropic and the condition $t_{\perp} \ll t_{\parallel}$ is satisfied. The Fermi surface under this condition becomes opened (see, Fig.b), consisting of two goffered lines. Note that a conventional Josephson coupling between the wires is realized under the condition $t_{\perp} < k_B T_{c0} \ll t_{\parallel}$, where T_{c0} is the critical temperature of the superconducting phase evaluated by means of the mean field theory.

We write Hamiltonian, given by Eqs.(1)-(3),

as $N \times N$ tridiagonal matrix in the basis of the generalized Nambu wave function of N superconducting wires,

$\psi_k^+ = (\psi_{1,k_x}^+, \psi_{2,k_x}^+, \dots, \psi_{j,k_x}^+, \dots, \psi_{N,k_x}^+)$ which is $4N$ dimensional vector with

$\psi_N^+ = (\psi_{j,\uparrow}^+(k_x), \psi_{j,\downarrow}^+(k_x), \dots, \psi_{j,\downarrow}^+(-k_x), -\psi_{j,\uparrow}^+(-k_x))$

for a single wire, $\hat{H} = \int \frac{dk_x}{2\pi} \psi_N^+ H_N \psi_N$. This expression can be exactly written in the two-wire Nambu basis,

$$\psi_j^{(2)+} = (\psi_{j,k_x}^+, \psi_{j+1,k_x}^+),$$

where $\hat{H} = \int \frac{dk_x}{2\pi} \sum_{j=1}^N \psi_N^{(2)+} H_j^{(2)} \psi_j^{(2)}$, (4)

$$H_j^{(2)} = \begin{pmatrix} H_{j,j} & H_{j,j+1} \\ H_{j+1,j} & H_{j+1,j+1} \end{pmatrix}, \quad (5)$$

and each $H_{j,j'}$ entry is 4×4 matrix,

$$H_{j,j} = \frac{1}{2} \{ \xi_{k_x} \tau_z + 2\alpha \sin k_x \tau_z \sigma_y +$$

$$+ 2\beta \sin k_x \tau_z \sigma_x + \varepsilon_z (\cos \theta_j \sigma_z +$$

$$+ \sin \theta_j \cos \phi_j \sigma_x +$$

$$+ \sin \theta_j \sin \phi_j \sigma_y) + |\Delta_0| \cos \phi_{0j} \tau_x -$$

$$- |\Delta_0| \sin \phi_{0j} \tau_y \}, \quad (6)$$

$$H_{j,j+1} = \frac{1}{2} \{ t_\perp \tau_z + i\bar{\alpha} \tau_z \sigma_x + i\bar{\beta} \tau_z \sigma_y +$$

$$+ |\Delta_1| \cos \phi_{1,j} \tau_x - |\Delta_1| \sin \phi_{1,j} \tau_y \} \quad (7)$$

Note that

$$H_{N,N+1} = H_{N+1,N} = 0 \text{ and } H_{N+1,N+1} \equiv H_{1,1} \text{ in Eq.}$$

(5) for $H_N^{(2)}$. The expressions (4)-(7) show that the BdG Hamiltonian written for N wires in the presence of the spin-orbit interactions and an external magnetic field can be reduced to the two-wires problem. Therefore, studies of problems, such as Josephson current, structures of the Abrikosov's vortices etc. for two-wires are enough

to extend the results to a realistic quasi-1D structure.

In the absence of the magnetic field, $B \equiv 0$, the time reversal invariance (TRI) is preserved, and a topological phase is shown to be realized in the superconducting state with intra-wire and inter-wire pairings. Hamiltonian, given by Eqs. (1)-(3), is transformed from discrete wire-number representation to the momentum space, and is written in the extended Nambu spinor basis

$$\psi_k^+ = (\psi_{k,\uparrow}^+, \psi_{k,\downarrow}^+, \psi_{-k,\downarrow}^+, -\psi_{-k,\uparrow}^+)$$

as $\hat{H} = \int \frac{dk_x}{2\pi} \int \frac{dk_y}{2\pi} \psi_k^+ H \psi_k$, where

$$H = \xi_{k_x} \tau_z + 2(\alpha \sin k_x - \bar{\beta} \sin k_y) \tau_z \sigma_y +$$

$$+ 2(\beta \sin k_x - \bar{\alpha} \sin k_y) \tau_z \sigma_x + \varepsilon_z (\cos \theta \sigma_z +$$

$$+ \sin \theta \cos \phi \sigma_x + \sin \theta \sin \phi \sigma_y) +$$

$$+ |\Delta| \cos \phi \tau_x - |\Delta| \sin \phi \tau_y, \quad (8)$$

where $\xi_k = -2t_{||} \cos k_x - 2t_{\perp} \cos k_y - \mu$ and ϕ is the total phase of the effective order parameter $\Delta(k_y) = \Delta_0 + 2\Delta_1 \cos k_y = |\Delta(k_y)| e^{i\phi}$. The Pauli matrices σ_i and τ_i act in the spin- and particle-hole spaces, respectively.

Time reversal symmetric topological superconductors. In the absence of the magnetic field and for real order parameter ($\phi = 0$), the time-reversal symmetry and the particle-hole symmetry are protected in topological SC. Hamiltonian $H(B=0) \equiv H_0$ satisfies the relations $\Theta H_0(k) = H_0(-k) \Theta$ with TRI operator $\Theta = i\tau_y K$ where K is the anti-unitary complex conjugation operator. The particle-hole symmetry emerges from the intrinsic structure of the BdG Hamiltonian, which relates quasi-particle excitations at $\pm E$ through the second quantized operator relation $\gamma_E = \gamma_{-E}^+$, signaling a formation of zero-mode Majorana state. This symmetry satisfies the anti-commutation relation $\Xi H_0(k) = -H_0(-k) \Xi$ with the particle-hole operator $\Xi = \tau_y \sigma_y K$,

obeying $\Xi^2 = -1$. The presence of time-reversal symmetry and particle-hole symmetry leads to a chiral symmetry $\Pi H_0(k) = -H_0(-k)\Pi$ where the unitary chiral operator Π is the product of Θ and Ξ , $\Pi = -i\Theta \cdot \Xi = \tau_y \sigma_0$.

The energy spectrum, obtained from $\det|E_0 - H_0| = 0$, reads

$$E_0 = s\sqrt{(\xi_k^2 \pm \varepsilon_s)^2 + \Delta^2(k_y)}, \quad (9)$$

where

$$\varepsilon_s = 2\sqrt{(\sin^2 k_x + \sin^2 k_y)(\alpha^2 + \beta^2) - 4\alpha\beta \sin k_x \sin k_y}$$

$$\text{with} \quad s = \pm \quad (10)$$

is the spin-orbit interactions energy, (we take $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta}$ for simplicity). The condition $\Delta(k_y) = 0$ determines the nodal points of the order parameter. $\Delta(k_y) = \Delta_0 + 2\Delta_1 \cos k_y$ changes sign at $\cos k_y = \mp \Delta_0 / (2\Delta_1)$ if $|\Delta_0| < 2\Delta_1$ as moving along k_y from $k_y = 0$ to $k_y = \pi$. A nontrivial TRI superconductor with $\nu = 1$ is realized if there are an odd number of Fermi surfaces with a negative pairing order parameter [3]. At the nodal points

$$E_{0N} = s[2t_{\parallel} \cos k_x + \mu - t_{\perp} \frac{\Delta_0}{2\Delta_1} \pm \varepsilon_{SN}(k_x, k_{yN})], \quad (11)$$

$$\text{where} \quad \delta = \sqrt{1 - \Delta_0^2 / (4\Delta_1^2)} \quad \text{and}$$

$\varepsilon_{SN}(k_x, k_{yN}) = 2\sqrt{(\sin^2 k_x + \delta^2)(\alpha^2 + \beta^2) - 4\alpha\beta\delta \sin k_x}$ is the value of the SOI energy, given by Eq. (10) at the nodal point $k_{yN} = \arccos(-\Delta_0 / (2\Delta_1))$. $\varepsilon_{SN}(k_x, k_{yN})$ varies between the maximal $\varepsilon_{SN}^{\max}$ and minimal $\varepsilon_{SN}^{\min}$ values. The order parameter switches sign as the nodal point is crossed. On the other hand, the SOIs split the Fermi surfaces.

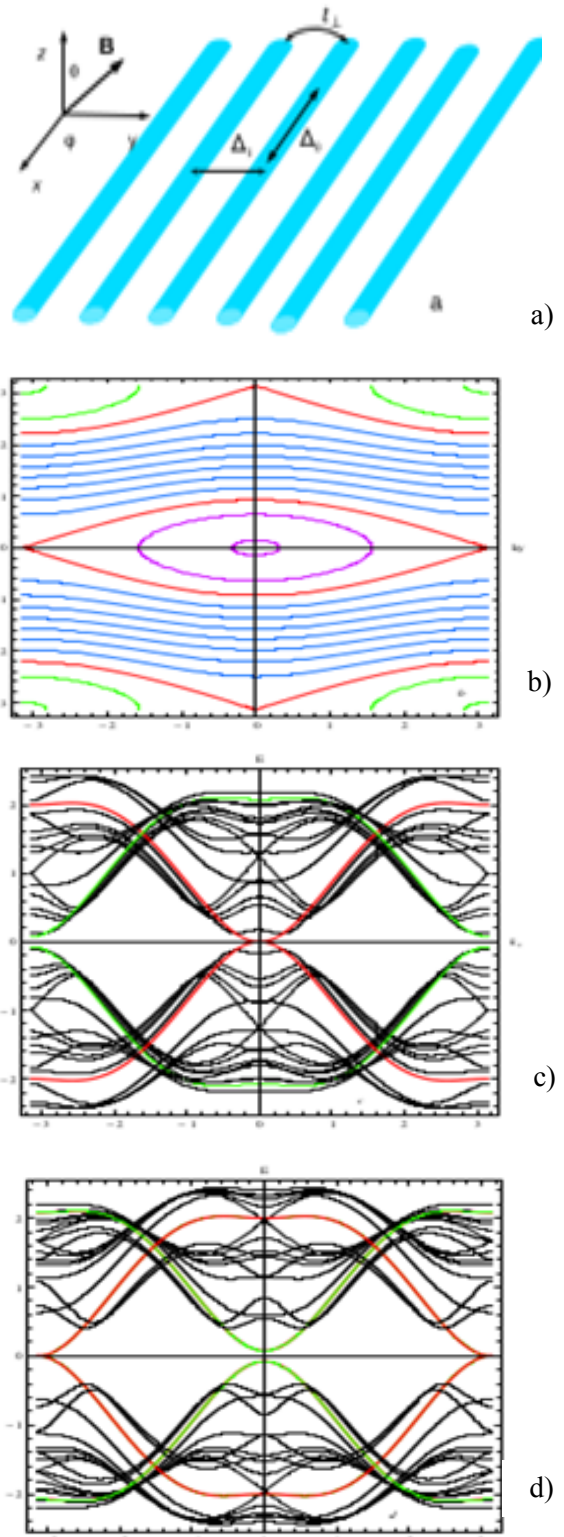


Figure. (a) Structure and (b) Fermi surface of quasi-1d superconducting wires on a plane. Energy spectrum for (c) and (d) $\Delta_0 > 0$ with a zero-energy state at the center of the Brillouin zone and $\Delta_0 < 0$ with a zero-energy state at the boundaries of the Brillouin zone

The splitted Fermi surfaces around the nodal points lie in the energetic interval $\varepsilon_{SN}(k_x, k_{yN})$ from each other. Non-trivial TRI topological phase with $\nu=1$ is realized when the maximal value of the kinetic energy term (the first three terms) in Eq. (11) is smaller than the minimal value of the SOIs mediated splitting energy

$$|2t_{//} + \mu - t_{\perp} \Delta_0 / \Delta_1| < \varepsilon_{SN}^{\min}(k_{x0}, \alpha, \beta).$$

The SC is fully gapped when $|2t_{//} + \mu - t_{\perp} \Delta_0 / \Delta_1| > \varepsilon_{SN}^{\max}(k_{x0}, \alpha, \beta).$

The calculation of the BdG quasi-particles' energy spectrum is simplified for $\beta=0$, for which $\varepsilon_{SN}^{\min}(k_{x0}, \alpha, \beta) = 2\alpha\delta$ and $\varepsilon_{SN}^{\max}(k_{x0}, \alpha, \beta) = 2\alpha\sqrt{1+\delta^2}$. The band structure of the topological SC with zero energy surface states for this case is drawn in figs. c, d. For $\Delta_0 < 0$, Majorana edge states appear at $k_x = 0$. The zero energy states move to the Brillouin zone boundaries, $k_x = \pm\pi$, for $\Delta_0 > 0$ (see, fig.d). The topologically non-trivial superconductor belongs to DIII symmetry class with Z_2 invariant, which takes a value $\nu=1$. In this case unpaired MFs at each end of a single wire form topologically protected Majorana Kramers pairs, yielding four zero-energy modes.

Conclusions. In this paper we argue that a TRI topological superconducting phase may realizes in the novel class of materials, consisting of regular, weakly-coupled superconducting wires in dielectric matrices [13,14]. The structures are fabricated under high pressures, which guarantee higher value of spin-orbit interactions. Experimentally observed enhancement of the critical temperature of these structures allows us to suggest that apart from the intra-wire pairing, the inter-wire pairing sets up too, yielding an effective (s+d) wave nodal order parameter. The order parameter changes sign by crossing the nodal point between two Fermi surfaces, splitted due to spin-orbit interactions. Note that the only requirement for re-

alization of a non-trivial topological superconductor is that the superconducting pair potential switches sign between the two Fermi surfaces. Time-reversal symmetric topological superconductor belongs to a DIII symmetry-class and is classified by the Z_2 topological invariant.

The distance between the superconducting wires in the cavities or channels of the dielectric matrices is enough large which allows us to detect the position of each wire under scanning tunneling microscope (STM). Topological phase in a single wire can be manipulated by supplying an electrical potential by means of the cantilever of the STM, which derives the wire from the superconducting phase to a normal metallic phase destroying Majorana quasi-particle in a particularly chosen wire.

REFERENCES

1. Hasan M. Z. and Kane C. L. Rev. Mod. Phys. 82, 30453067 (2010).
2. Schnyder A. P., Ryu S., Furusaki A., and A. W. W. Ludwig, Phys. Rev. B 78, 195125 (2008).
3. Qi X. L., Hughes T. L., and Zhang S. C., Phys. Rev. B 81, 134508 (2010).
4. Nayak C., Simon S. H., Stern A., Freedman M., and Das Sarma S., Rev. Mod. Phys. 80, 1083 (2008).
5. Moore G. and Read N., Nucl. Phys. B 360, 362 (1991).
6. Fu L. and Kane C. L., Phys. Rev. Lett. 100, 096407 (2008).
7. Sato M., Takahashi Y., and Fujimoto S., Phys. Rev. Lett. 103, 020401 (2009).
8. Lutchyn R. M., Sau J. D., and Das Sarma S., Phys. Rev. Lett. 105, 077001 (2010).
9. Oreg Y., Rafael G., and von Oppen F., Phys. Rev. Lett. 105, 177002 (2010).
10. Nakhmedov E. and Alekperov O., Phys. Rev. B 89, 195445 (2014).
11. Zhang F., Kane C. L., and Mele E. J., Phys. Rev. Lett. 111, 056402 (2013).
12. Wong C. L. M. and Law K. T., Phys. Rev. B 86, 184516 (2012).
13. Bogomolov V. N. and Kumzerov Yu. A., Pis'ma v Zh. Eksp. Teor. Fiz. 21, 434 (1975) [JETP Lett. 21, 198 (1975)].
14. Bogomolov V. N., Sov. Phys. Usp. 21, 77 (1978)[Usp. Fiz. Nauk 124, 171 (1978)].

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KVAZI-BİRÖLÇÜLÜ STRUKTURLARDA TOPOLOJİ İFRATKEÇİRİCİLƏR**Ə.P. Nəhmədov, S.O. Məmmədova, O.Z.Ələkbərov**

Kvazi-birölçülü ifratkeçirici materialların yeni sinfi təklif olunmuşdur ki, bu ifratkeçiricilər maqnit sahəsi olmadığı halda belə topoloji fazaya keçirlər. Bu tip zamanın tərs dəyişimində invariant qalan ifratkeçiricilər, seolit və asbest kimi dielektrlərdə periodik olaraq yerləşən boşluqlara ifratkeçirici maddələrin yüksək təzyiqlə doldurulması ilə alınır. Topoloji ifratkeçirici faza, yüksək spin-orbit təsirinə malik sistemlərdə ifratkeçirici naqillər boyunca s-simmetriyalı və naqillər arasında d-simmetriyalı Kuper cütlərinin yarandığı halda meydana çıxır. Bu halda, hər bir naqilin uclarında Majorana hallarının Kramers cütləri əmələ gəlir.

Açar sözlər: ifratkeçirici, Majorana fermionları, topoloji dielektrlər

ТОПОЛОГИЧЕСКАЯ СВЕРХПРОВОДИМОСТЬ В КВАЗИ-ОДНОМЕРНЫХ СТРУКТУРАХ**Э.П. Нахмедов, С.О. Мамедова, О.З.Алекперов**

Мы предложили новый класс сверхпроводящих квази-одномерных материалов, где топологическая фаза реализуется даже при отсутствии внешнего магнитного поля. Эти топологические сверхпроводники, которые инвариантны при обращении времени, изготавливаются с помощью внедрения сверхпроводящих материалов под высоким давлением в межатомные периодические каналы диэлектриков, таких как цеолит и асбест. Сверхпроводник переходит в топологическую фазу в присутствии большого значения спин-орбитального взаимодействия, когда Куперовские спаривания появляются как вдоль проволоки, также между соседними проволоками. Пары Крамерса Майорановских состояний возникают в концах каждой проволоки.

Ключевые слова: сверхпроводимость, Майорановские фермионы, топологические диэлектрики