

ON REPRESENTATION OF SOLUTIONS OF ROESSER TYPE DISCRETELY-CONTINUOUS LINEAR SYSTEM

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In the paper a boundary value problem for Roesser type discretely continuous linear system of equations is considered. The analogue of the Cauchy matrix is introduced, and the representation of the solution of the boundary value problem is found by its means.

Keywords: Roesser type system, boundary value problem, representation of the solution

The paper is devoted to finding the representation of the solution of a Roesser type linear, homogeneous hybrid system of equations.

1. Consider the Roesser type system of equations

$$\begin{aligned} z_i(t, x) &= A(t, x)z(t, x) + B(t, x)y(t, x) + f(t, x), \\ t \in T = [t_0, t_1], \quad x \in X = \{x_0, x_0 + 1, \dots, x_1\}, \quad (1) \\ y(t, x + 1) &= C(t, x)z(t, x) + D(t, x)y(t, x) + g(t, x), \\ t \in [t_0, t_1], \quad x = x_0, x_0 + 1, \dots, x_1 - 1, \end{aligned}$$

with the boundary conditions

$$z(t_0, x) = a(x), \quad x \in X, \quad y(t, x_0) = b(t). \quad (2)$$

Here $A(t, x)$, $B(t, x)$, $C(t, x)$, $D(t, x)$ are the given continuous with respect to t for all $x \in X$ matrix functions of appropriate dimensions, $(z(t, x), y(t, x))$ is the sought for $(n + m)$ -dimensional vector-function, $a(x)$ is the given n -dimensional discrete vector-function, $b(t)$ is the given m -dimensional continuous vector-function.

The system (1) consisting of the «mixture» of differential and difference equation is called Roesser-type hybrid system or a hybrid 2-D system. Many real processes are described by such systems (see e.i. [1, 2]).

In the paper we find «integral representation» of the solution of the system of equation (1) with boundary conditions (2).

2. Let

$$V(t, x; \tau, s) = \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix}$$

$n \times n$ $n \times m$
 $m \times n$ $m \times m$

is still unknown $(n + m) \times (n + m)$ dimensional block matrix function, $(z(t, x), y(t, x))$ is the solution of problem (1)-(2).

This means that

$$\begin{aligned} \begin{pmatrix} z_\tau(\tau + 1, s) \\ y(\tau, s + 1) \end{pmatrix} &= \\ &= \begin{pmatrix} A(\tau, s)z(\tau, s) + B(\tau, s)y(\tau, s) + f(\tau, s) \\ C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s) \end{pmatrix}. \end{aligned} \quad (3)$$

From (3) we get

$$\begin{aligned} \int_{t_0}^t \sum_{s=x_0}^{x-1} \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix} \begin{pmatrix} z_\tau(\tau, s) \\ y(\tau, s + 1) \end{pmatrix} d\tau = \\ = \int_{t_0}^t \sum_{s=x_0}^{x-1} \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix} \times \\ \times \begin{pmatrix} A(\tau, s)z(\tau, s) + B(\tau, s)y(\tau, s) + f(\tau, s) \\ C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s) \end{pmatrix} d\tau. \end{aligned} \quad (4)$$

From (4) using the rules of matrix multiplication will have:

$$\begin{aligned} \int_{t_0}^t \sum_{s=x_0}^{x-1} \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix} \begin{pmatrix} z_\tau(\tau, s) \\ y(\tau, s + 1) \end{pmatrix} d\tau = \\ = \int_{t_0}^t \sum_{s=x_0}^{x-1} \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix} [A(\tau, s)z(\tau, s) + B(\tau, s)y(\tau, s) + f(\tau, s)] + \\ + \begin{pmatrix} V_{11}(t, x; \tau, s) & V_{12}(t, x; \tau, s) \\ V_{21}(t, x; \tau, s) & V_{22}(t, x; \tau, s) \end{pmatrix} [C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s)] d\tau \end{aligned}$$

$$+V_{12}(t, x; \tau, s)[C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s)] \\ + V_{22}(t, x; \tau, s)[C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s)] \Big] d\tau.$$

On the basis of definition of equality of matrices we get the validity of the relations

$$\int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)z_\tau(\tau, s) + V_{12}(t, x; \tau, s)y(\tau, s+1)] d\tau = \\ = \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)[A(\tau, s)z(\tau, s) + B(\tau, s)y(\tau, s) + \\ + f(\tau, s)] + V_{12}(t, x; \tau, s)[C(\tau, s)z(\tau, s) + \\ + D(\tau, s)y(\tau, s) + g(\tau, s)] d\tau, \quad (5)$$

$$\int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{21}(t, x; \tau, s)z_\tau(\tau, s) + \\ + V_{22}(t, x; \tau, s)y(\tau, s+1)] d\tau = \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{21}(t, x; \tau, s) \times \\ \times [A(\tau, s)z(\tau, s) + B(\tau, s)y(\tau, s) + f(\tau, s)] + V_{22}(t, x; \tau, s) \times \\ \times [C(\tau, s)z(\tau, s) + D(\tau, s)y(\tau, s) + g(\tau, s)]] d\tau. \quad (6)$$

From (5), grouping the similar terms we get

$$\int_{t_0}^t \sum_{s=x_0}^{x-1} V_{11}(t, x; \tau, s)z_\tau(\tau, s) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s)y(\tau, s+1) d\tau = \\ = \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)A(\tau, s) + V_{12}(t, x; \tau, s)C(\tau, s)] \times \\ \times z(\tau, s) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)B(\tau, s) + V_{12}(t, x; \tau, s)D(\tau, s)] \times \\ \times y(\tau, s) d\tau + \\ + \int_{t_0}^t [V_{11}(t, x; \tau, s)f(\tau, s) + V_{12}(t, x; \tau, s)g(\tau, s)] d\tau.$$

In what follows, by using the formula of integration by parts in the certain integral, we can see the validity of the identity

$$\int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)z_\tau(\tau, s) + \\ + V_{12}(t, x; \tau, s)y(\tau, s+1)] d\tau = \sum_{s=x_0}^{x-1} V_{11}(t, x; t, s)z(t, s) -$$

$$- \sum_{s=x_0}^{x-1} V_{11}(t, x; t_0, s)z(t_0, s) - \\ - \int_{t_0}^t \sum_{s=x_0}^{x-1} \frac{\partial V_{11}(t, x; \tau, s)}{\partial \tau} z(\tau, s) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s-1)y(\tau, s) d\tau + \\ + \int_{t_0}^t V_{12}(t, x; \tau, x-1)y(\tau, x) d\tau - \\ - \int_{t_0}^t V_{12}(t, x; \tau, x_0-1)y(\tau, x_0) d\tau.$$

Consequently,

$$\sum_{s=x_0}^{x-1} V_{11}(t, x; t, s)z(t, s) - \sum_{s=x_0}^{x-1} V_{11}(t, x; t_0, s)z(t_0, s) - \\ - \int_{t_0}^t \sum_{s=x_0}^{x-1} \frac{\partial V_{11}(t, x; \tau, s)}{\partial \tau} z(\tau, s) d\tau + \\ + \int_{t_0}^t V_{12}(t, x; \tau, x-1)y(\tau, x) d\tau - \\ - \int_{t_0}^t V_{12}(t, x; \tau, x_0-1)y(\tau, x_0) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s-1)y(\tau, s) d\tau = \\ = \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)A(\tau, s) + V_{12}(t, x; \tau, s)C(\tau, s)] \times \\ \times z(\tau, s) d\tau + \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)B(\tau, s) + \\ + V_{12}(t, x; \tau, s)D(\tau, s)] y(\tau, s) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{11}(t, x; \tau, s)f(\tau, s) + V_{12}(t, x; \tau, s)g(\tau, s)] d\tau. \quad (7)$$

In the similar way, using (6) and discrete analogue of the formula of integration by parts, we get

$$\sum_{s=x_0}^{x-1} V_{21}(t, x; t, s)z(t, s) - \sum_{s=x_0}^{x-1} V_{21}(t, x; t_0, s)z(t_0, s) - \\ - \int_{t_0}^t \sum_{s=x_0}^{x-1} \frac{\partial V_{21}(t, x; \tau, s)}{\partial \tau} z(\tau, s) d\tau + \\ + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{22}(t, x; \tau, s-1)y(\tau, s) d\tau +$$

$$\begin{aligned}
& + \int_{t_0}^t V_{22}(t, x; \tau, x-1) y(\tau, x) d\tau - \\
& - \int_{t_0}^t V_{22}(t, x; \tau, x_0-1) y(\tau, x_0) d\tau = \\
& = \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{21}(t, x; \tau, s) A(\tau, s) + V_{22}(t, x; \tau, s) \times \\
& \times C(\tau, s)] z(\tau, s) d\tau + \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{21}(t, x; \tau, s) B(\tau, s) + \\
& + V_{22}(t, x; \tau, s) D(\tau, s)] y(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} [V_{21}(t, x; \tau, s) f(\tau, s) + V_{22}(t, x; \tau, s) g(\tau, s)] d\tau.
\end{aligned} \quad (8)$$

Assume that the matrix functions $V_{ij}(t, x; \tau, s)$, $i, j = 1, 2$ satisfy the relations

$$\begin{aligned}
& \frac{\partial V_{11}(t, x; \tau, s)}{\partial \tau} = \\
& = -V_{11}(t, x; \tau, s) A(\tau, s) - V_{12}(t, x; \tau, s) C(\tau, s), \\
& V_{12}(t, x; \tau, s-1) = V_{11}(t, x; \tau, s) B(\tau, s) + \\
& + V_{12}(t, x; \tau, s) D(\tau, s), \\
& \frac{\partial V_{21}(t, x; \tau, s)}{\partial \tau} = -V_{21}(t, x; \tau, s) A(\tau, s) - \\
& - V_{22}(t, x; \tau, s) C(\tau, s), \\
& V_{22}(t, x; \tau, s-1) = V_{21}(t, x; \tau, s) B(\tau, s) + \\
& + V_{22}(t, x; \tau, s) D(\tau, s), \\
& V_{11}(t, x; t, s) = 0, \quad x_0 \leq s \leq x-2, \\
& V_{11}(t, x; t, x-1) = E_1, \\
& V_{12}(t, x; \tau, x-1) = 0, \quad \tau \in [t_0, t], \\
& V_{21}(t, x; t, s) = 0, \quad x_0 \leq s \leq x-1, \\
& V_{22}(t, x; \tau, x-1) = E_2.
\end{aligned} \quad (9)$$

Then from (7) and (8) we get

$$\begin{aligned}
& \sum_{s=x_0}^{x-1} V_{11}(t, x; t, s) z(t, s) - \sum_{s=x_0}^{x-1} V_{11}(t, x; t_0, s) a(s) + \\
& + \int_{t_0}^t V_{12}(t, x; \tau, x-1) y(\tau, x) d\tau - \\
& - \int_{t_0}^t V_{12}(t, x; \tau, x_0-1) y(\tau, x_0) d\tau = \\
& = \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{11}(t, x; \tau, s) f(\tau, s) d\tau +
\end{aligned}$$

$$+ \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s) g(\tau, s) d\tau, \quad (11)$$

$$\begin{aligned}
& \sum_{s=x_0}^{x-1} V_{21}(t, x; t, s) z(t, s) - \sum_{s=x_0}^{x-1} V_{21}(t, x; t_0, s) z(t_0, s) + \\
& + \int_{t_0}^t V_{22}(t, x; \tau, x-1) y(\tau, x) d\tau - \\
& - \int_{t_0}^t V_{22}(t, x; \tau, x_0-1) y(\tau, x_0) d\tau = \\
& = \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{21}(t, x; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{22}(t, x; \tau, s) g(\tau, s) d\tau.
\end{aligned} \quad (12)$$

From (11) it follows that

$$\begin{aligned}
& \sum_{s=x_0}^{x-2} V_{11}(t, x; t, s) z(t, s) + V_{11}(t, x; t, x-1) z(t, x-1) - \\
& - \sum_{s=x_0}^{x-1} V_{11}(t, x; t_0, s) z(t_0, s) + \\
& + \int_{t_0}^t V_{12}(t, x; \tau, x-1) y(\tau, x) d\tau - \\
& - \int_{t_0}^t V_{12}(t, x; \tau, x_0-1) y(\tau, x_0) d\tau = \\
& = \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{11}(t, x; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s) g(\tau, s) d\tau.
\end{aligned}$$

Hence, allowing for (9), (10), we have:

$$\begin{aligned}
& z(t, x-1) = \sum_{s=x_0}^{x-1} V_{11}(t, x; t_0, s) z(t_0, s) + \\
& + \int_{t_0}^t V_{12}(t, x; \tau, x_0-1) y(\tau, x_0) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{11}(t, x; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x; \tau, s) g(\tau, s) d\tau,
\end{aligned}$$

that means

$$\begin{aligned}
z(t, x) = & \sum_{s=x_0}^x V_{11}(t, x+1; t_0, s) z(t_0, s) + \\
& + \int_{t_0}^t V_{12}(t, x+1; \tau, x_0-1) y(\tau, x_0) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^x V_{11}(t, x+1; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^x V_{12}(t, x+1; \tau, s) g(\tau, s) d\tau.
\end{aligned}$$

Thus, we get

$$\begin{aligned}
z(t, x) = & V_{11}(t, x+1; t_0, x) a(x) + \\
& + \sum_{s=x_0}^{x-1} V_{11}(t, x+1; t_0, s) a(s) + \\
& + \int_{t_0}^t V_{12}(t, x+1; \tau, x_0-1) b(\tau) d\tau + \\
& + \int_{t_0}^t V_{11}(t, x+1; \tau, x) f(\tau, x) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{11}(t, x+1; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{12}(t, x+1; \tau, s) g(\tau, s) d\tau.
\end{aligned} \quad (13)$$

In what follows, using identity (12), we get

$$\begin{aligned}
& \int_{t_0}^t V_{22}(t, x; \tau, x-1) y(\tau, x) d\tau = \\
& = \sum_{s=x_0}^{x-1} V_{21}(t, x; t_0, s) a(s) + \int_{t_0}^t V_{22}(t, x; \tau, x_0-1) \times \\
& \times y(\tau, x_0) d\tau + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{21}(t, x; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{22}(t, x; \tau, s) g(\tau, s) d\tau.
\end{aligned}$$

Hence, allowing for (9), (10) we get:

$$\begin{aligned}
\int_{t_0}^t y(\tau, x) d\tau = & \sum_{s=x_0}^{x-1} V_{21}(t, x; t_0, s) a(s) + \\
& + \int_{t_0}^t V_{22}(t, x; \tau, x_0-1) b(\tau) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{21}(t, x; \tau, s) f(\tau, s) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} V_{22}(t, x; \tau, s) g(\tau, s) d\tau.
\end{aligned}$$

Differentiating both parts of this relation with respect to t , we arrive at the representation

$$\begin{aligned}
y(t, x) = & \sum_{s=x_0}^{x-1} \frac{\partial V_{21}(t, x; t_0, s)}{\partial t} a(s) + \\
& + V_{22}(t, x; t, x_0-1) b(t) + \\
& + \int_{t_0}^t \frac{\partial V_{22}(t, x; \tau, x_0-1)}{\partial t} b(\tau) d\tau + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} \frac{\partial V_{21}(t, x; \tau, s)}{\partial t} f(\tau, s) d\tau + \\
& + \sum_{s=x_0}^{x-1} V_{22}(t, x; t, s) g(t, s) + \\
& + \int_{t_0}^t \sum_{s=x_0}^{x-1} \frac{\partial V_{22}(t, x; \tau, s)}{\partial t} g(\tau, s) d\tau.
\end{aligned} \quad (14)$$

Thus, we proved the following statement.

Theorem. Let, $V_{ij}(t, x; \tau, s)$, $i, j=1,2$ be matrix functions being the solutions of problem (9)-(10).

Then the solution of the system of equations (1), with boundary conditions (2) may be represented in the form (13), (14).

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ROSSER TIPLİ BİR XƏTTİ DİSKRET-KƏSİLMƏZ SİSTEMİN HƏLLİNİN GÖSTƏRİLİŞİ HAQQINDA

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Məqalədə Rosser tipli bir xətti bircins olmayan diskret-kəsilməz tənliklər sistemi üçün qoyulmuş sərhəd məsələsinə baxılır. Koşi matrisinin analoqu daxil edilmiş və onun vasitəsilə sərhəd məsələsinin həllinin göstərilişi tapılmışdır.

Açar sözlər: Rosser tipli sistem, sərhəd məsələsi, həllin göstərilişi

О ПРЕДСТАВЛЕНИИ РЕШЕНИЯ ОДНОЙ ДИСКРЕТНО-НЕПРЕРЫВНОЙ СИСТЕМЫ ТИПА РОССЕРА

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В статье изучается краевая задача для одной неоднородной линейной дискретно-непрерывной системы уравнений типа Россера. Введен аналог матрицы Коши, с помощью чего и найдено представление решения поставленной краевой задачи.

Ключевые слова: система типа Россера, краевая задача, представление решения