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Mathematics

OPTIMAL CONTROL SYSTEM OSCILLATIONS WITH ADDITIONAL RESTRICTIONS

A.J.Mamedov, B.M.Yusifov, H.H.Alieva

(Presented by Academician of NAS of Azerbaijan F.A.Aliev)

The paper considers the problem of reducing the system from the zero initial state to a given final state in the shortest possible time, with additional restrictions

Keywords: optimal control, functional, ℓ – time problems

Let controlled system described by the equation

$$\frac{\partial^2 y(x,t)}{\partial t^2} = a^2 \frac{\partial^2 y(x,t)}{\partial x^2} + f(x)u(t) \quad (1)$$

with the following initial and boundary conditions

$$y(x,0) = 0, \quad y_t'(x,0) = 0 \quad (2)$$

$$y(0,t) = 0, \quad y(l,t) = 0, \quad (3)$$

where l - the length of the rod $f(x)$ valid given function defined on $[0, \ell]$, $f(x) \neq 0$, $f(x) \in W_2^1(0, l)$, $u(t) \in L_2(0, t)$ - control actions.

Introduce additional constraints of the form:

$$E(u) = \int_0^T \left\{ \int_0^l \left[\left(\frac{\partial y(x,t)}{\partial t} \right)^2 + y^2(x,t) \right] dx + u^2(t) \right\} dt \leq L^2, \quad (4)$$

where $y(x,t)$ – generalized solution of (1) – (3) corresponding to the control $u(t)$.

Consider the problem: Among all the controls, satisfying the condition (4), find the one that takes the system (1), (3) from a given initial state (2) to a predetermined final state

$$y(x,T) = p(x), \quad y_t'(x,T) = g(x) \quad (5)$$

for possible minimum time T .

Here, $p(x) \in W_2^1(0, l)$ and $g(x)$ specified functions of $L_2(0, l)$.

Using the method of Fourier solution of the problem (1) - (3) for each fixed control $u(t) \in L_2(0, t)$ can be written as:

$$y(x,T) = \sum_{n=1}^{\infty} \frac{1}{an\pi} \int_0^t f_n u(\tau) \sin \frac{an\pi}{l} (t-\tau) d\tau \sin \frac{n\pi}{l} x \quad (6)$$

$$\text{where } f_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi}{l} x dx.$$

Using (6), as well as representation

$$p(x) = \sum_{n=1}^{\infty} p_n \int_0^t f_n u(\tau) \sin \frac{n\pi}{l} x,$$

$$p_n = \frac{2}{l} \int_0^l p(x) \sin \frac{n\pi}{l} x dx$$

$$g(x) = \sum_{n=1}^{\infty} g_n \sin \frac{n\pi}{l} x$$

$$g_n = \frac{2}{l} \int_0^l g(x) \sin \frac{n\pi}{l} x$$

and orthogonal system of functions $\left\{ \sin \frac{n\pi}{l} x \right\}$ for $[0, \ell]$ after some transformations:

$$\begin{cases} \sum_{n=1}^{\infty} \frac{1}{an\pi} f_n \int_0^T u(\tau) \sin \frac{an\pi}{l} (T-\tau) d\tau \sin \frac{n\pi}{l} x = \sum_{n=1}^{\infty} p_n \sin \frac{n\pi}{l} x \\ \sum_{n=1}^{\infty} f_n \int_0^T u(\tau) \cos \frac{an\pi}{l} (T-\tau) d\tau \sin \frac{n\pi}{l} x = \sum_{n=1}^{\infty} g_n \sin \frac{n\pi}{l} x \end{cases} \quad (7)$$

Now we show that the solution of the problem (1) - (3), (4), (5) can be reduced « L - the problem of moments" in the subspace $L_2(0, T)$.

Let $f(x) \in C^1[0, \ell]$. Given the formula (6), orthogonal system $\left\{ \sin \frac{n\pi}{l} x \right\}$ in $[0, \ell]$ functionality $E(u)$ can be written as:

$$E(u) = \int_0^T \left[\int_{\xi}^T H(t, \xi) u(t) d(t) + u(\xi) \right] u(\xi) d\xi \quad (8)$$

where

$$H(t, \xi) = \begin{cases} \frac{1}{2} \sum_{n=1}^{\infty} \left(1 + \frac{l^2}{a^2 n^2 \pi^2} \right) f_n^2 [(T - \xi) \cos \frac{an\pi}{l} (t - \xi) + \\ + 2 \cos \frac{an\pi}{l} (T - \xi) \sin \frac{an\pi}{l} (T - \xi)]; t \leq \xi, \\ \frac{1}{2} \sum_{n=1}^{\infty} \left(1 + \frac{l^2}{a^2 n^2 \pi^2} \right) f_n^2 [(T - t) \cos \frac{an\pi}{l} (\xi - t) + \\ + 2 \cos \frac{an\pi}{l} (T - \xi) \sin \frac{an\pi}{l} (T - t)]; t \geq \xi. \end{cases}$$

Now, in the space $L_2(0, T)$ introduce the scalar product of the elements as follows:

$$\langle \gamma_1(t), \gamma_2(t) \rangle = \int_0^T \left[\int_{\xi}^T H(t, \xi) \gamma_2(t) d(t) + \gamma_2(\xi) \right] \gamma_1(\xi) d\xi \quad (9)$$

$$\text{Let } h_{2m-1}(t) = \frac{1}{am\pi} f_m \cos \frac{am\pi}{l} t,$$

$$h_{2m}(t) = \frac{1}{an\pi} f_m \sin \frac{am\pi}{l} t,$$

$$\gamma_{2m-1}(T) = \alpha_m(T), \gamma_{2m}(T) = \beta_m(T), m = 1, 2, \dots$$

Then the system of equations (7) takes the following form:

$$\int_0^T h_k(t) u(t) d(t) = \gamma_k(T), \quad k = 1, 2, \dots \quad (10)$$

Next define the functions $p_k(t)$ as the solution of the auxiliary system of integral equations

$$\int_{\xi}^T H(t, \xi) p_k(t) d(t) + p_k(\xi) = h_k(\xi), \quad k = 1, 2, \dots \quad (11)$$

Now, using (11) the system of equations (10) written in the form:

$$\int_0^T \left[\int_{\xi}^T H(t, \xi) \rho_k(t) d(t) + \rho_k(\xi) \right] u(\xi) d\xi = \gamma_k(T), \quad k = 1, 2, \dots \quad (12)$$

So the solution to problem (1), (2), (3), (4) and (5) for any pair of $(p, g) \in K$ is equivalent to the following problem: find a function $u(t) \in L_2(0, T)$ and also the minimum time T that for any k the following conditions:

$$\int_0^T \left[\int_{\xi}^T H(t, \xi) \rho_k(t) d(t) + \rho_k(\xi) \right] u(\xi) d\xi = \gamma_k(T), \quad k = 1, 2, \dots \quad (13)$$

$$\int_0^T \left[\int_{\xi}^T H(t, \xi) u(t) d(t) + u(\xi) \right] u(\xi) d\xi \leq L^2 \quad (14)$$

Obviously, this task for any k is L - the moment problem in the space of $L_2(0, T)$, with the scalar product defined by the formula (9).

We have the following:

Theorem 1. In order to $L_2(0, T)$ in the space with the scalar product (10) there exists a function $u(t)$ from the norm not exceeding a positive number L and points sequence $\{\gamma_k(T)\}$ is necessary and sufficient that for all finite sets of numbers $\xi_1, \xi_2, \dots, \xi_{2m}$ inequality

$$\left[\sum_{k=1}^{2m} \xi_k \gamma_k(T) \right]^2 \leq L^2 \int_0^T \left[\sum_{k,n=1}^{2m} \xi_k \xi_n h_k(t) \rho_n(t) \right] u(\xi) d\xi \quad (15)$$

Let

$$Q_{kj}(T) = \int_0^T h_k(t) \rho_j(t) dt$$

Suppose

$$Q_n(T) = \sum_{k,j=1}^{2m} [L^2 Q_{kj}(T) - \gamma_k(T) \gamma_j(T)] \xi_k \xi_j$$

Then inequality (15) takes the form:

$$Q_{kj}(T) \geq 0 \quad (16)$$

Obviously, $Q_{kj}(T)$ is a quadratic form in the variables $\xi_1, \xi_2, \dots, \xi_{2m}$. Therefore, the condition (17) can be regarded as a condition of non-negativity of the quadratic form $Q_{kj}(T)$. As is known [4], for non-negative quadratic form necessary and sufficient that all the principal minors $A_{i_1, i_2, \dots, i_p}(T)$ it was non-negative matrix, that is,

$$A_{i_1, i_2, \dots, i_p}(T) \equiv A \begin{pmatrix} i_1 & i_2 & \dots & i_p \\ i_1 & i_2 & \dots & i_p \\ \dots & \dots & \dots & \dots \\ i_p & i_p & \dots & i_p \end{pmatrix} (T) = \begin{vmatrix} a_{i_1 i_1} & a_{i_1 i_2} & \dots & a_{i_1 i_p} \\ a_{i_2 i_1} & a_{i_2 i_2} & \dots & a_{i_2 i_p} \\ \dots & \dots & \dots & \dots \\ a_{i_p i_1} & a_{i_p i_2} & \dots & a_{i_p i_p} \end{vmatrix} \geq 0$$

Define the set of $J_{i_1, i_2, \dots, i_p}$ and J_n follows:

$$J_{i_1, i_2, \dots, i_p} = \{T \mid A_{i_1, i_2, \dots, i_p} \geq 0\}$$

$$J_n = \bigcap_{1 \leq i_1 < i_2 < \dots < i_p \leq 2m}$$

Clear that $J_{n+1} \subset J_n$.

It is easy to prove the following theorem.

Theorem 2. a) If, for any set of n J_n is not empty, then the problem (1) - (3), (4), (5) has a solution with the minimum amount of time T^0 equal $T^0 = \lim_{n \rightarrow \infty} T_n$, where $T_n = \inf J_n$.

b) If for some set of n J_n empty, then the problem (1) - (3), (4), (5) has no solution.

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Sumgayit State University
akpammammadov@rambler.ru

SİSTEMİN RƏQSİNİN ƏLAVƏ MƏHDUDİYYƏTLİ ƏN TEZ OPTİMAL İDARƏETMƏ MƏSƏLƏSİ

Ə.C.Məmmədov, B.M.Yusifov, H.H.Əliyeva

Məqalədə rəqs edən sistemi əlavə məhdudiyyət daxilində verilmiş başlangıç vəziyyətdən son vəziyyətə ən tez gətirən optimal idarəedicinin varlığı və yeganəliyi isbat olunur.

Açar sözlər: optimal idarəedici, funksional, ℓ - moment problemi

ОПТИМАЛЬНОЕ ПО БЫСТРОДЕЙСТВИЮ УПРАВЛЕНИЕ КОЛЕБАНИЯ СИСТЕМЫ С ДОПОЛНИТЕЛЬНЫМИ ОГРАНИЧЕНИЯМИ

А.Д.Мамедов, Б.М.Юсифов, Х.Г.Алиева

В работе рассмотрена задача о приведении системы из нулевого начального состояния в заданное конечное состояние за наименьшее время, при дополнительных ограничениях. Доказана теорема о существовании и единственности.

Ключевые слова: оптимальное управление, функционал, ℓ - проблема моментов