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FINDING THE FRACTIONAL ORDER OF THE OSCILLATOR SYSTEM BY THE ASYMPTOTIC METHOD IN THE DISCRETE CASE

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In practice, for example, at the production of oil by a rod pump, the movement of the spring is described by either differential or difference equations, and then, an asymptotic separation is obtained to determine the fractional order in discrete conditions.

Keywords: oscillator system, fractional subtraction derivative parameter, second-order Volterra integral equation, golden division method, quadratic functional, least squares method.

Introduction

We know that the motion of oscillation systems inside a fluid is described by the following second-order differential equation with fractional derivatives [1-5,10]:

$$my''(x) + aD^\alpha y(x) + by(x) = f(x), \quad x > 0, \quad \alpha \in (1,2), \quad (1)$$

and an initial condition

$$\begin{cases} y(0) = 0 \\ y'(0) = y_{10} \end{cases} \quad (2)$$



Fig.1. The motion of oscillation systems inside a fluid

where $a=2s\sqrt{\eta\rho}$, $b=k$, m is mass, s is a hard plate area, ρ is the density, μ is Newtonian fluid viscosity, k is spring constant, $f(x)$ is external force. Let's determine the substitution $\frac{1}{m} = \varepsilon$. Given that our equation includes a small parameter, then the problem we are considering will be in the form (2)-(3).

$$y''(x) + \varepsilon a D^\alpha y(x) + \varepsilon b y(x) = \varepsilon f(x), \quad x > 0, \quad \alpha \in (1,2), \quad (3)$$

where a, b and y_{10} are constants, $\varepsilon > 0$ is a small parameter, $\alpha \in (1,2)$, $f(x)$ is continuous real uncton, $y(x)$ is desired function. We bring the Cauchy problem to the II type Volterra integral equation[6-9]:

$$y''(x) + \int_0^x K_\alpha(x-t)y''(t)dt = F(x), \quad x > 0, \quad (4)$$

where

$$K_\alpha(x-t) = \varepsilon \left\{ a \frac{(x-t)^{1-\alpha}}{(1-\alpha)!} + b(x-t) \right\} \quad (5)$$

$$F(x) = \varepsilon \left\{ f(x) - \left[a \frac{x^{1-\alpha}}{(1-\alpha)!} + bx \right] y_{10} \right\} \quad (6)$$

Discretization algorithm. Discretize equation (1), (2) for even and odd index we have:

$$\begin{aligned} y_{2k} = & (2 - hK_\alpha(x_{2k-2} - x_{2k-3}))y_{2k-1} + (-1 - 2hK_\alpha(x_{2k-2} - x_{2k-3}) - \\ & - hK_\alpha(x_{2k-2} - x_{2k-4}))y_{2k-2} - \\ & - h \sum_{m=2}^{2k-3} (K_\alpha(x_{2k-2} - x_{m-2}) - 2K_\alpha(x_{2k-2} - x_{m-1}) + K_\alpha(x_{2k-2} - x_m))y_m + \\ & + 2hK_\alpha(x_{2k-2} - x_0)y_1 - K_\alpha(x_{2k-2} - x_0)y_0 - \\ & - hK_\alpha(x_{2k-2} - x_1)y_1 + h^2 F_{2k-1}(\varepsilon) \end{aligned} \quad (7)$$

$$\begin{aligned}
y_{2k+1} = & (3 - 2hK_\alpha(x_{2k-2} - x_{2k-3}) - 2hK_\alpha(x_{2k-1} - x_{2k-2}))y_{2k-1} + \\
& + (-2 + 4hK_\alpha(x_{2k-2} - x_{2k-3}) - 2hK_\alpha(x_{2k-2} - x_{2k-4}) - \\
& - K_\alpha(x_{2k-1} - x_{2k-4}) + 2hK_\alpha(x_{2k-1} - x_{2k-3}))y_{2k-2} - \\
& - h \sum_{m=2}^{2k-3} 2K_\alpha(x_{2k-2} - x_{m-2}) - 4K_\alpha(x_{2k-2} - x_{m-1}) + \\
& + 2K_\alpha(x_{2k-2} - x_m) + K_\alpha(x_{2k-1} - x_{m-2}) - 2K_\alpha(x_{2k-1} - x_{m-1}) + \\
& + K_\alpha(x_{2k-1} - x_m)y_m + h(4K_\alpha(x_{2k-2} - x_0) - 2K_\alpha(x_{2k-2} - x_1) + \\
& + 2K_\alpha(x_{2k-1} - x_0) - K_\alpha(x_{2k-1} - x_1))y_1 - \\
& h(2K_\alpha(x_{2k-2} - x_0) + K_\alpha(x_{2k-1} - x_0))y_0 + 2h^2F_{2k-2}(\varepsilon) + \\
& h^2F_{2k-1}(\varepsilon), k > 1
\end{aligned} \quad (8)$$

$$W_k = \begin{pmatrix} y_{2k} \\ y_{2k+1} \end{pmatrix} k \geq 0 \quad (9)$$

The system of algebraic equations (7) and (8) is collected in W_k . If we consider them in (9), we obtain

$$\begin{aligned}
W_k = & A^{(k-1)}(\varepsilon)W_{k-1} + \sum_{j=0}^{k-2} A^{(j)}(\varepsilon)W_j + \\
& + h^2 \tilde{F}_n(\varepsilon), k \geq 1
\end{aligned} \quad (10)$$

where

$$A^{(j)}(\varepsilon) = \begin{bmatrix} A_{11}^{(j)}(\varepsilon) & A_{12}^{(j)}(\varepsilon) \\ A_{21}^{(j)}(\varepsilon) & A_{22}^{(j)}(\varepsilon) \end{bmatrix} \quad (11)$$

$$\begin{aligned}
A_{11}^{(k-1)}(\varepsilon) = & -1 + 2hK_\alpha(x_{2k-2} - x_{2k-3}) - \\
& - hK_\alpha(x_{2k-2} - x_{2k-4}),
\end{aligned} \quad (12)$$

$$A_{12}^{(k-1)}(\varepsilon) = 2 - hK_\alpha(x_{2k-2} - x_{2k-3}), \quad (13)$$

$$\begin{aligned}
A_{21}^{(k-1)}(\varepsilon) = & 2 + 4hK_\alpha(x_{2k-2} - x_{2k-3}) - \\
& - 2hK_\alpha(x_{2k-2} - x_{2k-4}) - hK_\alpha(x_{2k-1} - x_{2k-4}) + \\
& + 2hK_\alpha(x_{2k-1} - x_{2k-3}),
\end{aligned} \quad (14)$$

$$\begin{aligned}
A_{22}^{(k-1)}(\varepsilon) = & 3 - 2hK_\alpha(x_{2k-2} - x_{2k-3}) - \\
& - hK_\alpha(x_{2k-1} - x_{2k-3})
\end{aligned} \quad (15)$$

$$\begin{aligned}
A_{11}^{(kj)}(\varepsilon) = & -h[K_\alpha(x_{2k-2} - x_{2j-2}) - 2K_\alpha(x_{2k-2} - x_{2j-1}) + \\
& + K_\alpha(x_{2k-2} - x_{2j})], j = \overline{1, k-2},
\end{aligned} \quad (16)$$

$$\begin{aligned}
A_{12}^{(k)}(\varepsilon) = & -h[K_\alpha(x_{2k-2} - x_{2j-1}) - 2K_\alpha(x_{2k-2} - x_{2j}) + \\
& + K_\alpha(x_{2k-2} - x_{2j+1})], j = \overline{1, k-2},
\end{aligned} \quad (17)$$

$$\begin{aligned}
A_{21}^{(k)}(\varepsilon) = & -h[2K_\alpha(x_{2k-2} - x_{2j-2}) - 4K_\alpha(x_{2k-2} - x_{2j-1}) + \\
& + 2K_\alpha(x_{2k-2} - x_{2j}) + K_\alpha(x_{2k-1} - x_{2j-2}) - \\
& - 2K_\alpha(x_{2k-1} - x_{2j-1}) + K_\alpha(x_{2k-1} - x_{2j})], j = \overline{1, k-2},
\end{aligned} \quad (18)$$

$$\begin{aligned}
A_{22}^{(k)}(\varepsilon) = & -h[2K_\alpha(x_{2k-2} - x_{2j-1}) - 4K_\alpha(x_{2k-2} - x_{2j}) + \\
& + 2K_\alpha(x_{2k-2} - x_{2j+1}) + K_\alpha(x_{2k-1} - x_{2j-1}) - \\
& - 2K_\alpha(x_{2k-1} - x_{2j}) + K_\alpha(x_{2k-1} - x_{2j+1})], j = \overline{1, k-2}.
\end{aligned} \quad (19)$$

$$A_{11}^{(0)}(\varepsilon) = -hK_\alpha(x_{2k-2} - x_0), \quad (20)$$

$$A_{12}^{(0)}(\varepsilon) = 2hK_\alpha(x_{2k-2} - x_0) - hK_\alpha(x_{2k-2} - x_1), \quad (21)$$

$$A_{21}^{(0)}(\varepsilon) = -h[2K_\alpha(x_{2k-2} - x_0) + K_\alpha(x_{2k-1} - x_0)], \quad (22)$$

$$\begin{aligned}
A_{22}^{(0)}(\varepsilon) = & h \left[4K_\alpha(x_{2k-2} - x_0) - 2K_\alpha(x_{2k-2} - x_1) + \right. \\
& \left. + 2hK_\alpha(x_{2k-1} - x_0) - K_\alpha(x_{2k-1} - x_1) \right]
\end{aligned} \quad (23)$$

$$\begin{aligned}
\tilde{F}_{k1}(\varepsilon) = & F_{2k-1}(\varepsilon) = \varepsilon \left\{ f_{2k-1} - \left[a \frac{x_{2k-1}}{(1-\alpha)!} + bx_{2k-1} \right] y_{10} \right\}
\end{aligned} \quad (24)$$

$$\begin{aligned}
\tilde{F}_{k2}(\varepsilon) = & 2F_{2k-2}(\varepsilon) + F_{2k-1}(\varepsilon) = \\
& 2\varepsilon \left\{ f_{2k-2} - \left[a \frac{x_{2k-2}}{(1-\alpha)!} + bx_{2k-2} \right] y_{10} \right\} + \\
& + \varepsilon \left\{ f_{2k-1} - \left[a \frac{x_{2k-1}}{(1-\alpha)!} + bx_{2k-1} \right] y_{10} \right\}
\end{aligned} \quad (25)$$

$$\begin{aligned}
W_n = & W_n(\varepsilon) = A^0(\varepsilon)W_0 + \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix} A^0(\varepsilon)W_0 + \\
& + \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix} \tilde{F}_{n-1}(\varepsilon) + \tilde{F}_n(\varepsilon) = W_{0n} + \varepsilon W_{1n}
\end{aligned} \quad (26)$$

$$W_{0n} = 0, \quad W_{1n} = \frac{W_n(\varepsilon)}{\varepsilon} \quad (27)$$

$$\begin{aligned}
& + 2h\epsilon\alpha \left[\left((x_{2k-2} - x_0) \ln^2(x_{2k-2} - x_0) - (x_{2k-2} - x_0) \ln(x_{2k-2} - x_0) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-2} - x_0) \ln(x_{2k-2} - x_0) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-2} - x_0) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_1 - \\
& - 6h\epsilon\alpha \left[\left((x_{2k-2} - x_1) \ln^2(x_{2k-2} - x_1) - (x_{2k-2} - x_1) \ln(x_{2k-2} - x_1) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-2} - x_1) \ln(x_{2k-2} - x_1) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-2} - x_1) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_1 + \\
& + 8h\epsilon\alpha \left[\left((x_{2k-1} - x_0) \ln^2(x_{2k-1} - x_0) - (x_{2k-1} - x_0) \ln(x_{2k-1} - x_0) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-1} - x_0) \ln(x_{2k-1} - x_0) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-1} - x_0) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_1 - \\
& - 4h\epsilon\alpha \left[\left((x_{2k-1} - x_1) \ln^2(x_{2k-1} - x_1) - (x_{2k-1} - x_1) \ln(x_{2k-1} - x_1) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-1} - x_1) \ln(x_{2k-1} - x_1) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-1} - x_1) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_1 - \\
& - 2\epsilon \left[\left((x_{2k-3}) \ln^2(x_{2k-3}) - (x_{2k-3}) \ln(x_{2k-3}) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-3}) \ln(x_{2k-3}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-3}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_{10} + 6\epsilon \left[\left((x_{2k-4}) \ln^2(x_{2k-4}) - (x_{2k-4}) \ln(x_{2k-4}) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-4}) \ln(x_{2k-4}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-4}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_{10} + \\
& + 3\epsilon \left[\left((x_{2k-3}) \ln^2(x_{2k-3}) - (x_{2k-3}) \ln(x_{2k-3}) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-3}) \ln(x_{2k-3}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-3}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_{10} + 2\epsilon \left[\left((x_{2k-2}) \ln^2(x_{2k-2}) - (x_{2k-2}) \ln(x_{2k-2}) \right) \left(-\int_0^\infty e^{-t} \ln t dt \right) - \left((x_{2k-2}) \ln(x_{2k-2}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-2}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_{10} + \\
& + \mathcal{E} \left\{ \left[\left((x_{2k-1}) \ln(x_{2k-1}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-1}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) - \left((x_{2k-1}) \ln(x_{2k-1}) \int_0^\infty e^{-t} \ln t dt + \left(-(x_{2k-1}) \int_0^\infty e^{-t} \ln^2 t dt \right) \right) \right] y_{10} \right\} \\
& - \left[\left((x_{2k-1}) \ln(x_{2k-1}) \int_0^\infty e^{-t} \ln t dt \right) \left((-2) \int_0^\infty e^{-t} \ln t dt \right) \right]
\end{aligned}
\tag{36}$$

Then the expressions (29) and (30) get the following simple form:

$$J_1 = - \sum_{n=0}^{k-1} (W_{1n}' RW_n^{st} + W_n^{st} RW_{1n}) \quad (41)$$

Separation of α with α_0 and α_1

If we take W_n^{st} a column vector consisting of any fixed numbers $\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$ and take the separation of α in the form $(\alpha_0 + \varepsilon\alpha_1)$ and found:

Construction of quadratic functional

Given the method of least squares, we can write the corresponding quadratic function as follows:

$$J(\varepsilon) = \sum_{n=0}^{k-1} (W_n(\varepsilon) - W_n^{st})' R (W_n(\varepsilon) - W_n^{st}) = \\ -\varepsilon \sum_{n=0}^{k-1} W_{1n}' W_n^{st} - \varepsilon \sum_{n=0}^{k-1} W_n^{st}' R W_{1n} + \sum_{n=0}^{k-1} W_n^{st}' R W_n^{st} = J_0 + \varepsilon J_1 \quad (39)$$

$$J_0 = \sum_{n=0}^{k-1} W_n^{st'} R W_n^{st} \quad (40)$$

$$\sum_{n=0}^{k-1} \begin{pmatrix} \varphi_1 + (\alpha_0 + \varepsilon \alpha_1) \varphi_2 \\ \psi_1 + (\alpha_0 + \varepsilon \alpha_1) \psi_2 \end{pmatrix}' R \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} + \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}' R \begin{pmatrix} \varphi_1 + (\alpha_0 + \varepsilon \alpha_1) \varphi_2 \\ \psi_1 + (\alpha_0 + \varepsilon \alpha_1) \psi_2 \end{pmatrix} = 0$$

$$\sum_{n=0}^{k-1} \begin{pmatrix} \varepsilon \varphi_1 + \varepsilon (\alpha_0 + \varepsilon \alpha_1) \varphi_2 \\ \varepsilon \psi_1 + \varepsilon (\alpha_0 + \varepsilon \alpha_1) \psi_2 \end{pmatrix}' R \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} + \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}' R \begin{pmatrix} \varepsilon \varphi_1 + \varepsilon (\alpha_0 + \varepsilon \alpha_1) \varphi_2 \\ \varepsilon \psi_1 + \varepsilon (\alpha_0 + \varepsilon \alpha_1) \psi_2 \end{pmatrix} = 0$$

Therefore, in the system of equations differs only in terms of the parameter ε .

$$\begin{aligned} & \chi_1 \varphi_1 + \chi_1 (\alpha_0 + \varepsilon \alpha_1) \varphi_2 + \chi_2 \psi_1 + \chi_2 (\alpha_0 + \varepsilon \alpha_1) \psi_2 + \\ & + \chi_1 \varphi_1 + \chi_1 (\alpha_0 + \varepsilon \alpha_1) \varphi_2 + \chi_2 \psi_1 + \chi_2 (\alpha_0 + \varepsilon \alpha_1) \psi_2 = 0 \end{aligned}$$

$$\alpha_0 + \varepsilon \alpha_1 = \frac{\chi_1 \varphi_1 + \chi_2 \psi_1}{\chi_1 \varphi_2 + \chi_2 \psi_2} \quad (42)$$

Consider the expression (42) as a linear separation $\alpha_0 = \frac{\chi_1 \varphi_1 + \chi_2 \psi_1}{\chi_1 \varphi_2 + \chi_2 \psi_2}$ $\alpha_1 = 0$ then $\alpha = \alpha_0$.

According to, for n:

$$\begin{aligned} & \alpha_0 + \varepsilon \alpha_1 + \varepsilon^2 \alpha_2 + \varepsilon^3 \alpha_3 + \dots + \varepsilon^n \alpha_n + \dots \\ & \alpha_0 + \varepsilon \alpha_1 + \varepsilon^2 \alpha_2 + \varepsilon^3 \alpha_3 + \dots + \varepsilon^n \alpha_n + \dots \\ & = \frac{\chi_1 \varphi_1 + \chi_2 \psi_1}{\chi_1 \varphi_2 + \chi_2 \psi_2} = 0 \\ & \alpha_1 = 0, \alpha_2 = 0, \dots, \alpha_n = 0, \dots \end{aligned}$$

Result. So the α found in the given problem is α_0 . Thus, we can propose the following algorithm:

The values of the parameters a, b, f and the number of l, n -distribution points are given.

$x \in [0.1, 1]$ and $\alpha \in (1, 2)$ and ε - the value of the smallest parameter of the problem is given.

An initial condition $y(0) = 0, y_{10} = 1$ is included

For each $n, W_n(\alpha), W_{1n}(\alpha) j = 1, 2k-1$ calculated by formulas (26) and (27).

According to formulas (39)-(41) $J_0(\alpha), J_1(\alpha)$ and $J(\alpha)$ calculated.

Using the golden division method, we select the condition $\|J(\alpha^*) - J(\alpha)\| < \varepsilon_1$ and checked it. If the condition is met, the process stops, otherwise proceed to step 5.

Let's check the result on an example.

Example: Let's solve the example given in [1] by entering a small parameter.

1. In the problem (2),(3) $a = 3, b = 1, f = 8$.

2. Distribution interval of $x \in [0.1, 1], n = 10,$

$$l = 5, h = \frac{1-0.1}{n}, \Delta\alpha = \frac{1.9-1.1}{l}, \varepsilon = 10^{-6}$$

3. $W_n(\alpha), W_{1n}(\alpha) j = 1, 2k-1, J_0(\alpha), J_1(\alpha)$ and $J(\alpha)$ calculate.

4. $\alpha^* \alpha_0 = 1.1$ we solve the problem by continuing the calculation process until the following condition α_k is satisfied using the golden rule: $|\alpha_k - \alpha_{k-1}| < \varepsilon_1$ and found $\alpha^* = 1.67192$

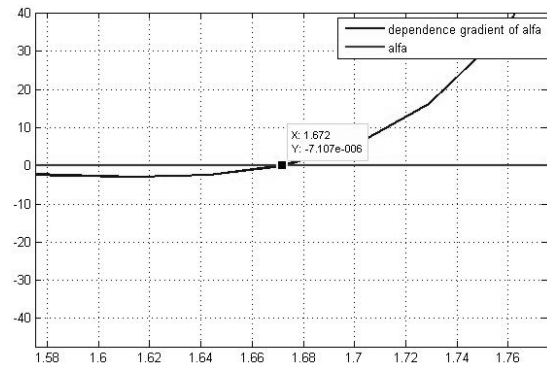


Fig.2. Dependence gradient of alfa

The results obtained by changing the number of distribution points and the value of the small parameter ε are given in the table below.

Table 1

Distribution points of x	Distribution points of α	Received α^*	Small parameter
10	5	1.7487	$10^{(-1)}$
		1.6870	$10^{(-2)}$
		1.6737	$10^{(-3)}$
		1.6721	$10^{(-4)}$
		1.6719	$10^{(-5)}$
		1.6719	$10^{(-6)}$
50	10	1.5841	$10^{(-1)}$
		1.5918	$10^{(-2)}$
		1.5945	$10^{(-3)}$
		1.5949	$10^{(-4)}$
		1.5949	$10^{(-5)}$
		1.5949	$10^{(-6)}$

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OSİLLATOR SİSTEMİN DİSKRET HALINDA ASİMPTOTİK ÜSUL İLƏ KƏSR TƏRTİBİN TAPILMASI

A.F. Rəsulzadə

Praktikada elə məsələlər var ki, məsələn, ştanq nasos ilə neft hasilində, prunjerin hərəkəti ya diferensial, ya da fərq tənlikləri ilə təsvir edilir. Qoyulmuş məsələyə ε kiçik parametr daxil edərək asimptotik ayrılışı vermək üçün əvvəlcə kəsr tərtib törəmənin analitik təsviri verilir və bu zaman intervallarında hərəkət tənlikləri diskretləşdirilir sonda isə diskret şəraitdə kəsr tərtibin təyini üçün asimptotik ayrılış alınır.

Açar sözlər: *ossillator sistemi, kəsr tərtib törəmə, kiçik parametr, ikinci tərtibli Volterra integral tənliyi, qızıl bölgü üsulu, kvadratik funksional, ən kiçik kvadratlara üsulu*

НАХОЖДЕНИЕ ДРОБНОГО ПОРЯДКА СИСТЕМЫ ОСЦИЛЛЯТОРОВ АСИМПТОТИЧЕСКИМ МЕТОДОМ В ДИСКРЕТНОМ СЛУЧАЕ

А.Ф. Расулзаде

На практике, например, при добыче нефти штанговым насосом движение пружины описывается либо дифференциальными, либо разностными уравнениями дискретно, и в итоге получается асимптотическое разделение для определения дробного порядка в дискретных условиях.

Ключевые слова: *система осцилляторов, параметр производной дробного вычитания, интегральное уравнение Вольтерра второго порядка, метод золотого деления, квадратичный функционал, метод наименьших квадратов*