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ADDITIONAL ANALYSIS FOR APPLYING THE TISSERAND INVARIANT IN SOLVING THE PLANET X PROBLEM

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The article provides an additional analysis of the Tisserand's constant distribution of 1389 long-period comets. Planet X as perturbation body is considered according to the hypothesis of one of the authors. The previously obtained results regarding the redundancy of values in the range 2.5-3 are confirmed. The most significant correspondence is established in the case of the use of "barycentric" elements of orbits

Keywords: long-period comets, Planet X, Tisserand's constant, statistics

Introduction

This paper is a continuation of the work [1], where the Tisserand criterion in different variants is applied to the testing of some hypotheses about an unknown massive planet in the Solar System. The object of testing was mainly Guliyev's hypothesis [2]. The original form of the Tisserand constant is:

$$T = \frac{1}{a} + 2 \frac{\sqrt{a_p}}{r} \sqrt{a(1-e^2)} \cos i \quad (1)$$

where a , e and i are elements of the cometary orbit, a_p and r are the semi-major axis and heliocentric distance of the planet at the moment of the putative perturbation of the cometary orbit in the sphere of influence, respectively. For a long period of time the value of this constant remains unchanged.

In the cited work we used the method of "gluing" the orbit of Planet X. In other words, at various segments of the orbit is considered as circular with semi-major axis r . Then the Tisserand invariant looks as follows:

$$T = \frac{r(1-e)}{q} + 2 \sqrt{\frac{q(1+e)}{r}} \cos i' \quad (2)$$

In this case, the value of i' is defined with respect to the plane of motion of the planet. In [1] it is shown that the number of T values in the optimal range from 2.5 to 3 or near it is excessive. The calculations use the heliocentric

distance of the planet in the directions of the aphelion and the distant node of the cometary orbit as the parameter r . The statistics of the parameter T was carried out on the basis of three arrays of comets: all known comets; elliptical comets and comets with known "original" barycentric orbits. All cases revealed an excess of the T values.

The problem formulation with the data used

In this paper, all calculation procedures will be based on the classical formula (1), instead of (2). This approach has the following purposes: to exclude the possibility of obtaining positive effects in the distribution of T by replacing the elliptical orbit of the planet by a set of circular orbits. In the case of application of the standard variant of the criterion, we may obtain less efficient results. However, they should satisfy the requirements of mathematical statistics. Proceeding in this way, we push again one or another hypothesis of an unknown planet to additional validation.

In the calculations, the distance r will be computed using the formulae:

$$r = \frac{1-e_p^2}{1+e_p \cos(\omega_p + L' + 180^\circ)} \quad \text{and} \quad r = \frac{1-e_p^2}{1+e_p \cos(\omega_p + \Omega')} \quad (3)$$

here L' and Ω' are the longitude of perihelion and the ascending node of the cometary orbit relative to the plane where its ascending node is the reference point, ip and ω_p – are the angular elements of the cometary orbit in this frame, ep – is the eccentricity of the putative orbit of planet. The first of these formulae yields the heliocentric distance of the planet in the direction of the aphelion of comet, and the second in the direction of the distant node.

In work xxx, we used data of 1389 long-period comets, including 635 elliptical comets discovered up to mid-2020. In the present work is able to expand this list to 1437 due to new comets. It starts and ends with comets 1P/-239 K1 and C/2021 O1, respectively. The principles of the list are given in [2]. Herewith the number of elliptical comets increased to 672. The list of comets with "original" barycentric orbits remains unchanged (488 comets). It is important to note that the number of T values in the range 2.04 – 2.98 increased by 5 after the comets added. The increase was due to elliptical comets.

Calculation results

The calculations were performed using the same pattern that was used in [1]. The heliocentric distance r of the planet was specified for two scenarios: when the probable interaction

on between the comet and the planet occurs in the direction of the aphelion of the cometary orbit; when it takes place in the direction of the distant node of the cometary orbit. After calculating the value of the parameter T , similar computations were carried out with respect to 67 virtual planets. The comparison was realized using the Student's t -distribution.

To this end, we have determined the mean value of n for the virtual planets, its standard deviation σ , the normed difference $t = (N-n)/\sigma$, and its confidence probability α .

The calculation results are given in Table. For the total set of comets the correlation of the statistical data (second line of the table) with the hypothesis of an unknown planet with parameters:

$$ap = 339 \text{ AU } (\pm 34); ep = 0.16(\pm 0.02); \omega_p = 570(\pm 15); \Omega_p = 2710.7(\pm 3); ip = 860 (\pm 2)$$

in both cases is good. The statistical dataset for elliptical comets (line 4) provides a slightly inferior result, since the value of α in this case does not reach 0.95. However, if we use the data set for comets with known barycentric "original" orbits [3], the resulted α even exceeds 0.999. The set of statistical data for this case is reflected on line 5 of the table.

Table

Comparative frequencies of perihelia relative to the solar apex and other directions

Calculation method for r	By the aphelia of comet orbits						By the distant nodes of comet orbits					
Comet Group Composition	Range for T	N	n	σ	t	α	Range for T	N	n	σ	t	α
Total set	2.7-2.85	7	3.55	1.09	3.16	>0.99	2.7-2.85	7	3.64	1.19	2.82	>0.99
Elliptical	2.70-2.85	7	4.97	1.47	1.38	>0.91	2.70-2.85	7	5.89	1.42	1.49	>0.93
Barycentric	2.43- 2.78	7	3.58	0.78	4.37	>0.99	2.43- 2.78	7	3.64	0.77	4.35	>0.99

Conclusion

The calculations demonstrated that the use of the classical form of the Tisserand criterion and its testing on virtual planes is well consistent with the hypothesis under the study. We should emphasize the importance of the analysis for the barycentric "original" cometary orbits. As it is known, they are mostly free from the influence of the giant planets. Consequently, we could expect the most principal results from them within the framework of the problem. The statistical sets for this class of comets completely exclude the random factor.

Therefore, in addition to the results of [1] we can once again argue that the distribution of the Tisserand constant is well-aligned

with the provisions of the studied hypothesis of the unknown planet. Other available hypotheses can also be investigated in a similar way.

REFERENCES

1. Guliyev A.S., Nabyev Sh., Guliyev R.A. Test of the Hypothesis for an Unknown Distant Massive Planet in the Solar System Applying Tisserand's Criterion to a System of Long-Period Comets // Earth, Moon and Planets, 2021, vol. 125, iss. 2, pp. 1-9.
2. Guliyev A.S., Guliyev R.A. System of Long-Period Comets as Indicator of the Large Planetary Body on the Periphery of the Solar System // Acta Astronomica, 2019, vol. 69, iss. 2, pp. 177-20.
3. Williams G.V., Marsden B.G. Catalogue of Cometary Orbits. 17-th edition, 195p. (2008)

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TISSERAN İNVARİANTININ TƏTBİQİ İLƏ X PLANETİ PROBLEMİNİN TƏDQIQİ ÜZRƏ ƏLAVƏ TƏHLİL

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Məqalədə 1389 uzun periodlu kometin Tisseran sabiti qiymətlərinin əlavə təhlili yerinə yetirilmişdir. Həyəcanlandırıcı cisim qismində müəlliflərdən birinə mənsub hipotezindəki X planeti nəzərdə tutulur. Bundan əvvəl yerinə yetirilən təhlildəki 2.5-3 intervalındakı qiymət bolluğu faktı öz təsdiqini tapmışdır. Ən nəzərəçarpan uyğunluq "barisentrik" elementlərin istifadə olunduğu halda alınmışdır.

Açar sözlər: uzunperiodlu kometlər, X Planeti, Tisseran meyarı, statistika

ДОПОЛНИТЕЛЬНЫЙ АНАЛИЗ ПРИМЕНЕНИЯ ИНВАРИАНТА ТИССЕРАНА В РЕШЕНИИ ПРОБЛЕМЫ ПЛАНЕТЫ X

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В статье проведен дополнительный анализ распределения константы Тиссерана 1389 долгопериодических комет. В качестве возмущающего тела рассматривается Планета X по гипотезе одного из авторов. Подтверждены ранее полученные результаты относительно избыточности значений в интервале 2.5-3. Наиболее значимое соответствие установлено в случае использования «барицентрических» элементов орбит.

Ключевые слова: долгопериодические кометы, Планета X, критерий Тиссерана, статистика