

UDC 004.942:519.711.3

ALGORITHM FOR SOLUTION OF LINEAR QUADRATIC OPTIMIZATION PROBLEM WITH CONSTRAINT IN THE FORM OF EQUALITIES ON CONTROL IN A CERTAIN PART OF THE TIME INTERVAL

N.S. Hajiyeval¹, M.M. Mutallimov^{1,2}, Academician of ANAS F.A. Aliev^{1,2}

In this paper the linear quadratic optimization problem is considered. In current problem there is a CONSTRAINT in the form of equalities on the control actions in a known part of the time interval. Constructing extended functional, the corresponding Euler-Lagrange equations are described. Solving these equations, control actions and the corresponding trajectory on each part of the time interval are constructed, i.e. on the part where there is no limitation on control and vertical external force, on the part the control has a limitation. The results are illustrated by the example of vertical movement and hovering of a flying object at a certain height under action.

Keywords: linear quadratic optimization problem, Euler-Lagrange equation, control action, optimal program trajectory, the system of linear algebraic equation

Introduction

Recently, many problems for constructing program trajectories and control [1-3] are reduced to solution linear quadratic optimization problem with corresponding boundary conditions. In this case, the solution method is based on constructing the corresponding extended functional [3-6] and describing the Euler-Lagrange equations with finding the boundary conditions for the Lagrange multipliers. Solving the Euler-Lagrange equations with boundary conditions on phase coordinates and Lagrange multipliers, the corresponding control action and coordinates of the object are constructed.

But when some coordinates of control actions have CONSTRAINTs in the form of equalities, then the similar algorithms do not work and the corresponding modification is required, where in the current work a corresponding modified algorithm is developed. The results are illustrated using a specific example of a vertical quadcopter with curtains at the appropriate height for a certain time.

Problem Statement

Let the movement of the object in the time interval $[0, T + \Delta]$ is described by the following linear differential equation

$$\dot{P}(t) = FP(t) + Gu(t) + V, \quad (1)$$

where $P(t) = [z_1(t) \ z_2(t) \ z_3(t) \ z_4(t)]'$ – phase coordinates of n -dimensional object, constant matrices F and G are controlled pairs with dimensions $n \times n$, $n \times m$, respectively. V is an external disturbance with dimension $n \times 1$. $u(t) = [u_1(t) \ u_2(t)]'$ – vector of control actions. Let denote $G = [G_1 \ G_2]$.

Assume that there is the following CONSTRAINT for $u_1(t)$ in the time interval $(T + 0, T + \Delta)$

$$u_1(t) = C, \quad T + 0 < t \leq T + \Delta, \quad (2)$$

and boundary conditions for the phase coordinates have the form

$$z_1(0) = z_0, \ z_1(T - 0) = z_d, \ z_3(0) = z_{30}, \ z_4(0) = 0, \quad (3)$$

where C , z_0 , z_d , z_{30} are constant vectors.

It is required to find control action $u(t)$ with CONSTRAINT (2) and the corresponding trajectory $P(t)$ with boundary conditions (3), such that the equation (1) is satisfied and the following functional received the minimum value

$$J = \frac{1}{2} z'_2(T-0)c_1z_2(T-0) + \frac{1}{2} z'_3(T-0)c_2z_3(T-0) + \frac{1}{2} z'_4(T-0)c_3z_4(T-0) + \frac{1}{2} \int_0^{T+\Delta} u'(t)Ru(t)dt, \quad (4)$$

i.e. linear quadratic optimization problem (1)-(4) with minimal control actions is fulfilled, where c_1, c_2, c_3 are matrices with corresponding dimensions, R is a symmetric matrix with corresponding dimension.

Euler-Lagrange equation for the problem (1)-(4) and construction solution on the interval $[0, T + \Delta]$

Taking into account the CONSTRAINT (2) in (1) and (4), we get that on the interval $0 \leq t < T-0$ the equation of the object is written as (1), on the interval $T+0 < t \leq T+\Delta$ the equation of the object is written as

$$\dot{P}(t) = FP(t) + G_2u_2(t) + G_1C + V, \quad (5)$$

and the functional (4) will be as follows

$$J = \frac{1}{2} z'_2(T-0)c_1z_2(T-0) + \frac{1}{2} z'_3(T-0)c_2z_3(T-0) + \frac{1}{2} z'_4(T-0)c_3z_4(T-0) + \frac{1}{2} \int_0^T u'(t)R_1u(t)dt + \frac{1}{2} \int_T^{T+\Delta} u'_2(t)R_2u_2(t)dt, \quad (6)$$

where R_1 is a symmetric matrix with corresponding dimension, R_2 is a known parameter.

Thus, using the results of [3, 4] we have the Euler-Lagrange equations on the intervals $[0, T-0), (T+0, T+\Delta]$ for the system (1), (5) and the functional (6) in the following form, correspondingly

$$\begin{bmatrix} \dot{P}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} F & -GR_1^{-1}G' \\ 0 & -F' \end{bmatrix} \begin{bmatrix} P(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} V \\ 0 \end{bmatrix}, \quad 0 \leq t < T-0, \quad (7)$$

$$\begin{bmatrix} \dot{P}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} F & -G_2R_2^{-1}G'_2 \\ 0 & -F' \end{bmatrix} \begin{bmatrix} P(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} G_1C + V \\ 0 \end{bmatrix}, \quad T+0 < t \leq T+\Delta, \quad (8)$$

where $\lambda(t)$ is a Lagrange multiplier with corresponding dimension.

Then using the results of [7-12] for $\lambda(t)$ we have the following boundary conditions for the system (7)

$$\lambda_2(0) = 0, c_1z_2(T-0) - \lambda_2(T-0) = 0, c_2z_3(T-0) - \lambda_3(T-0) = 0, c_3z_4(T-0) - \lambda_4(T-0) = 0. \quad (9)$$

Note that the condition (9) together (3) is sufficient to restore $P(t)$, $\lambda(t)$ on the interval $[0, T-0)$.

The control $u(t)$ on the interval $[0, T-0)$ is defined as

$$u(t) = -R^{-1}G'\lambda(t), \quad (10)$$

and the control $u_2(t)$ on the interval $(T+0, T+\Delta]$ is defined as

$$u_2(t) = -R_2^{-1}(G_2'^2)'\lambda(t). \quad (11)$$

Assume that

$$F = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, G_1 = \begin{bmatrix} G_1^1 \\ 0 \end{bmatrix}, G_2 = \begin{bmatrix} 0 \\ G_2^2 \end{bmatrix}, V = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}. \quad (12)$$

Then from (8) we get that $z(t) = [z_1(t) \ z_2(t)]'$ is defined by the following system of differential equations on the interval $(T+0, T+\Delta]$

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \\ \dot{z}_3(t) \\ \dot{z}_4(t) \end{bmatrix} = \begin{bmatrix} F_1 & 0 \\ 0 & -F_1 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \end{bmatrix} + \begin{bmatrix} G_1^1C + V_1 \\ 0 \end{bmatrix} \quad (13)$$

with boundary conditions

$$z_1(T+0) = z_{1T}, z_2(T+0) = z_{2T}, \lambda_1(T+\Delta) = 0, \lambda_2(T+\Delta) = 0, \quad (14)$$

where the first and the second conditions have been found from (7).

$z_3(t), z_4(t)$ is defined from the following differential equations

$$\begin{bmatrix} \dot{z}_3(t) \\ \dot{z}_4(t) \\ \dot{\lambda}_3(t) \\ \dot{\lambda}_4(t) \end{bmatrix} = \begin{bmatrix} F_2 & -G_2^2R_2^{-1}(G_2^2)' \\ 0 & -F_2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \\ \lambda_3(t) \\ \lambda_4(t) \end{bmatrix} \quad (15)$$

with boundary conditions

$$z_3(T+0) = z_{3T}, z_4(T+0) = z_{4T}, \lambda_3(T+\Delta) = 0, \lambda_4(T+\Delta) = 0, \quad (16)$$

where the first and the second conditions have been found from (7).

Note that the initial conditions (16) are sufficient to find $z_3(t)$, $z_4(t)$, $\lambda_3(t)$, $\lambda_4(t)$ on the interval $(T, T+\Delta]$.

Thus, solving the system of equations (7) with boundary conditions (3), (9) we get $x(t)$, $\lambda(t)$ and the control $u(t)$ is defined from (10) on the interval $[0, T-0)$. Then solving the systems of equations (13), (15) with initial conditions (14), (16), respectively we get $x(t)$, $\lambda(t)$ and the control $u_2(t)$ is defined from (11) on the interval $(T+0, T+\Delta]$.

Example

Let consider an example that describes the vertical movements of a flying object to a given height and at a certain time Δ remains at the same height. For simplicity, we will assume that the object moves only vertically and does not perform any rotation relative to the vertical axis. In this case, equation (1) will have the following form

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = F \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + Gu(t) + V, \quad 0 \leq t \leq T + \Delta \quad (17)$$

where

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, V = \begin{bmatrix} 0 \\ -g \end{bmatrix}, g \text{ is an}$$

acceleration velocity.

The boundary conditions (3) for $P(t) = [z_1(t) \ z_2(t)]^T$ don't change. Then the problem (1)-(4) is reduced to the following problem

$$\dot{P}(t) = FP(t) + Gu(t) + V, \quad 0 \leq t \leq T + \Delta, \quad (18)$$

$$z_1(0) = z_0, \quad z_1(T-0) = z_d, \quad (19)$$

$$u(t) = C, \quad T+0 < t \leq T + \Delta, \quad (20)$$

$$J = \frac{1}{2} c_1 z_2^2(T-0) + \frac{1}{2} \int_0^{T+\Delta} u^2(t) dt \rightarrow \min. \quad (21)$$

Using the procedure outlined in paragraphs 2-4, we can find P (Fig.1) solving equations (7), (13), (15) on the intervals $[0, T-0)$, $(T+0, T+\Delta]$ and the control action $u(t)$ (Fig.2) is determined by formula (10) on the interval $[0, T-0)$.

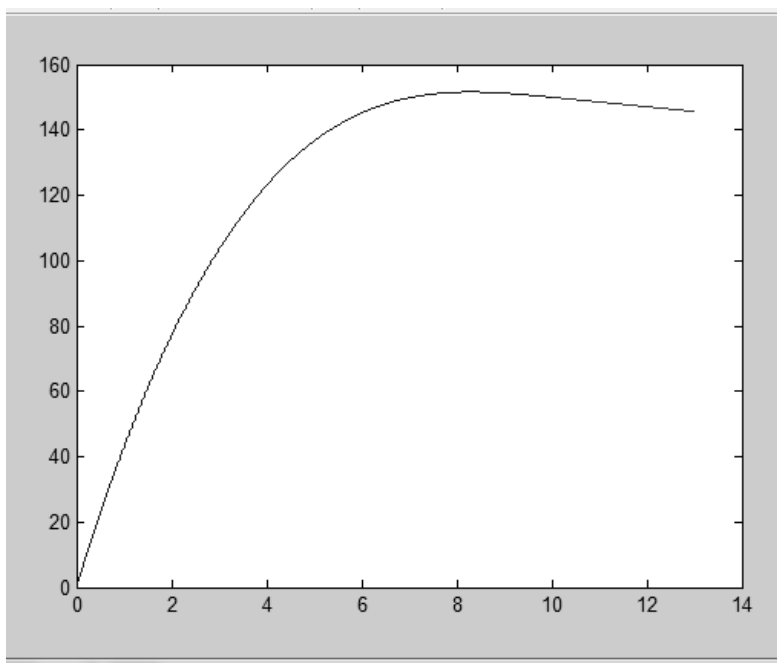


Fig.1. Changing $z_1(t)$ on the interval $[0, T + \Delta]$

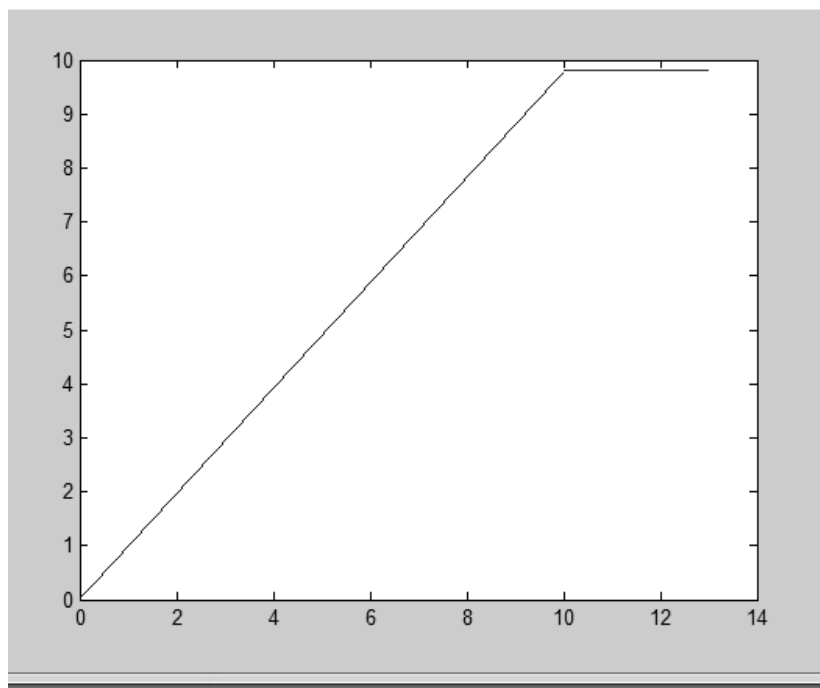


Fig.2. Changing $u(t)$ on the interval $[0, T + \Delta]$

REFERENCES

1. *Larin V.B., Tunik A.A. Ilynska S.I.* Some algorithms for unmanned aerial vehicles navigation, Outskirts, 2020, 204 p.
2. *Aliev F.A., Larin V.B.* Optimization of Linear Control System. London: Gordon and Breach, 1998, 270 p.
3. *Алиев Ф.А.* Методы решения прикладных задач оптимизации динамических систем. Баку: Елм, 1989, 320 с.
4. *Брайсон А., Хо Ю Ши.* Прикладная теория оптимального управления. М: Мир, 1972.
5. *Алиев Ф.А., Бордюг Б.А., Ларин В.Б.* Спектральный метод решения матричных алгебраических уравнений Риккати // Докл. АН СССР, 1987, 292:4, с.783–788.
6. *Муталлимов М.М., Алиев Ф.А.* Методы решения задач оптимизации при эксплуатации нефтяных скважин. Saarbrücken (Deutschland): LAP LAMBERT, 2012, 164 с.
7. *Алиев Ф.А.* Задача оптимального управления линейной системой с неразделенными двухточечными

краевыми условиями. // Дифференциальные уравнения, 1986, № 2, с. 345-347.

8. *Алиев Ф.А.* Задача оптимизации с двухточечными краевыми условиями. // Известия АН СССР, сер.техн. кибернетика, 1985, № 6, с. 138-146.

9. *Муталлимов М.М., Гаджиева Н.С.* Построение оптимальных программных траекторий и управлений при вертикальном движении квадрокоптера // Proceedings of IAM, V.12, N.2, 2023, pp.166-180.

10. *Aliev F.A., Mutallimov M.M., Velieva N.I., Safarova N.A., Jafarov A.G.* Algorithms for solving the stabilization problems of the quadcopter motion by GPS data // Applied and Computational Mathematics, V.22, N.4, 2023, pp.520-527.

11. *Алиев Ф.А., Джавадов Г.Н., Муталлимов М.М.* Проблемы стабилизации движением квадрокоптера по данным GPS // НАНА, Доклады, 79 (1-2), pp.10-16.

12. *Aliev F.A., Sushchenko O.A., Mutallimov M.M., Javadov N.G., Mammadov F. F., Maharramov R.R.* Algorithm for quadcopter motion stabilization taking into account data of inertial navigation system // TWMS J. Pure Appl. Math., V.14, N.2, 2023, pp.278-289.

¹*Institute of Applied Mathematics, Baku State University*

²*Institute of Information Technology of the Ministry of Science and Education of Azerbaijan*
f_aliev@yahoo.com

ZAMAN İNTERVALININ MÜƏYYƏN HİSSƏSİNDƏ İDARƏEDİCİYƏ BƏRABƏRLİK ŞƏKLİNDƏ MƏHDUDLUQ QOYULDUQDA XƏTTİ KVADRATİK OPTİMALLAŞMA MƏSƏLƏSİNİN HƏLLİ ALQORİTMİ

N.S. Hacıyeva, M.M.Mütəllibov, F.Ə.Əliyev

Bu işdə zaman intervalının məlum hissəsində idarəedici təsir üzərində bərabərliklər şəklində məhdudiyyətlər olduğu halda xətti kvadratik optimallaşdırma problemi nəzərdən keçirilir. Genişləndirilmiş funksionalı tərtib etməklə müvafiq Eyler-Laqranj tənlikləri təsvir edilir və onları həll etməklə zaman intervalının hər iki hissəsi üçün idarəedici təsir və müvafiq trayektoriya qurulur, yəni idarəediciyə heç bir məhdudiyyətin olmadığı hissədə və idarəediciyə məhdudiyyətlərin olduğu hissədə həll tapılır. Nəticələr şaquli xarici qüvvənin təsiri altında uçan cismin müəyyən hündürlüyə şaquli hərəkəti və müəyyən məsafədə asılı qalması misalına baxılmışdır.

Açar sözlər: xətti kvadratik optimallaşdırma məsələsi, Eyler-Laqranj tənliyi, idarəedici təsir, optimal trayektoriya, xətti cəbri tənliklər sistemi

АЛГОРИТМ РЕШЕНИЯ ЗАДАЧИ ЛИНЕЙНОЙ КВАДРАТИЧНОЙ ОПТИМИЗАЦИИ С ОГРАНИЧЕНИЕМ В ВИДЕ РАВЕНСТВ ПО УПРАВЛЕНИЮ НА ОПРЕДЕЛЕННОЙ ЧАСТИ ВРЕМЕННОГО ИНТЕРВАЛА

Н.С. Гаджиева, М.М. Муталлимов, Ф.А.Алиев

В данной работе рассматривается задача линейно-квадратичной оптимизации, где на известной части интервала времени имеются ограничения в виде равенств на управляющие воздействия. Путем составления расширенного функционала описываются соответствующие уравнения Эйлера-Лагранжа, где путем их решения строятся управляющие воздействия и соответствующая траектория для каждой части интервала времени, т.е. на участках, где нет ограничений на управление и на участках, где элемент управления имеет ограничения. Результаты проиллюстрированы на примере вертикального движения и зависания летающего объекта на определенной высоте под действием вертикальной внешней силы.

Ключевые слова: задачи линейно-квадратичной оптимизации, уравнение Эйлера-Лагранжа, управляющее воздействие, оптимальная программная траектория, система линейных алгебраических уравнений