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# INTERVALS OF STABILITY AND RELIABILITY OF THE EARTH'S MODEL PARAMETERS

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*It is shown that deformations have concrete intervals of continuous change within the framework of which conditions of mechanics of deformable solid body on determination the parameters of the medium are maintained. Critical values of deformations corresponding to various process of loss of stability determine the boundaries of these intervals.*

***Keywords:*** *the Earth's theoretical models, finite deformation, density, stability of equilibrium state*

**Introduction**

Problems of determining the domains of applicability of various tectonophysical parameters are reduced to the evaluation of limits of change of homogeneous and uniform deformations of volumetric compression from the standpoint of three-dimensional theory of elastic stability of equilibrium states [1-6].

**Confidence interval on deformations. "Internal" instability.** In case of uniform compression by nonconservative ("tracing") external load, the elastic equilibrium state of the deformed isotropic body (within the compressed model of the medium) is stable [6] irrespective of its geometric shape in the interval

$$\lambda_1^* < \lambda_1 < 1 \tag{1}$$

In case of modeling of deformation process using harmonic elastic potential [3,6]

$$\lambda_1^* = \frac{1+\nu}{2-\nu}\ , \tag{2}$$

$\lambda_1$ is elongation along the coordinate axes; $\nu$ is Poisson's ratio.

$0 < \nu < 0.5$ is known. Consequently, we obtain the value $0.5 < \lambda_1^* < 1$ for ultimate elongation $\lambda_1^*$ for all possible materials. In case of homogeneous initial finite deformations [6]

$$\lambda_1^2 = 1 + 2\varepsilon_0 \tag{3}$$

we determine the domain of its change using formulae (2) and (3) for the deformation parameter

$$\varepsilon_0^* < \varepsilon_0 < 0\,;\ \ \varepsilon_0^* = \frac{3}{2}\frac{2\nu-1}{(2-\nu)^2}\,, \tag{4}$$

where the equilibrium state is stable.

In case of quadratic elastic potential $\lambda_1^*$ is defined in the form [3,6]

$$\lambda_1^* = \left(\frac{1+\nu}{2-\nu}\right)^{\frac{1}{2}}\ . \tag{5}$$

Using formulae (3) and (5) we define the critical parameter of deformation

$$\varepsilon_0^* = \frac{1}{2}\frac{2\nu-1}{2-\nu}\,. \tag{6}$$

The domain of stability is also defined as the inequality (4) considering (6) in this case.

We obtain the following equations on the basis of these results to define possible intervals of density change of the Earth's material in cases of harmonic and quadratic elastic potentials of finite deformations respectively

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)^*;\ \left(\frac{\rho}{\rho_0}\right)^* = \left(1+2\varepsilon_0^*\right)^{-\frac{3}{2}} = \left(\frac{2-\nu}{1+\nu}\right)^3, \tag{7}$$

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)^*; \quad \left(\frac{\rho}{\rho_0}\right)^* = \left(\frac{2-\nu}{1+\nu}\right)^{\frac{3}{2}}. \quad (8)$$

The achievement of shortening and deformations of their critical values (2), (5) and (4), (6) corresponds to "internal" instability [6]. These values of deformations determine the theoretical limit of durability of the considered materials [6]. Consolidation process of the medium is changed into deconsolidation process in achieving this critical value of deformation.

Thus, experimental and observation data should be applied to describe density distribution and state equation of the Earth's material maintaining the condition imposed on deformation $\varepsilon_0^* < \varepsilon_0 < 0$.

**Instability of deformation on geometric forming.** The inequations mentioned above allow theoretically determining the possible limits of variation of the studied parameters within the accepted conditions. These intervals may be different in actual practice due to the properties of deformation process even at the elastic stage. If there are inhomogeneities of physico-mechanical and geometric origin in the medium, more complex deformation processes can occur in it.

Let's consider the case of homogeneous deformation of the medium influenced by conservative ("dead") external loads. The state of elastic equilibrium body loses the stability at lower values $(\varepsilon_0)_*$ than critical deformations $\varepsilon_0^*$ at such impact. Let's consider the stability of elastic equilibrium state of half-space in the neighborhoods of vertical cylindrical cavity of circular cross-section as a specific example. The "dead" loads are defined on the cylindrical surface of the cavity, intensity of which is equal to the value of intensity of the external load acting on the "infinity" along horizontal planes. In this case, homogeneous deformed state is realized in the neighborhoods of the cavity [9].

It is shown that the state of elastic equilibrium is unstable in the neighborhoods of the cylindrical cavity in case of influencing on its surface by conservative forces. The following analytical expressions are obtained for critical values of elongation $\lambda_3 = \lambda_2 = (\lambda_1)_*$ corresponding to the loss of stability of the equilibrium state with geometric forming:
in case of harmonic potential

$$(\lambda_1)_* = -\frac{(1+\nu)(1+2\nu)}{2(2+\nu-4\nu^2)}\left\{1-\left[1+\frac{8(2+\nu-4\nu^2)}{(1+2\nu)^2}\right]^{\frac{1}{2}}\right\}; \quad (9)$$

in case of quadratic potential

$$(\lambda_1)_* = \left(\frac{3}{3-2x}\right)^{\frac{1}{2}};$$

$$x = -\frac{3(5-4\nu)}{16(1+\nu)}\left\{1-\left[1-\frac{16(1-2\nu)}{(5-4\nu)^2}\right]^{\frac{1}{2}}\right\}. \quad (10)$$

The corresponding values of critical deformations $(\varepsilon_0)_*$ are determined using the formula (3) considering (9) and (10). The body loses the stability and gets the geometric shape through the change of the existed geometric shape with more stable state of equilibrium in achieving the values of deformations of critical values $(\varepsilon_0)_*$. The distribution of stress and deformations becomes nonuniform in new state.

Numerical results are shown in Table for critical values of shortening and deformations at various data of Poisson's ratio calculated on formulae (2), (4)-(6), (9) and (10). The results corresponding to harmonic are given in the numerator, and quadratic in the denominations. It follows from these results that the loss of stability of elastic equilibrium state caused by "internal" instability occurs in the neighborhood of the cylindrical cavity in this case. Stable intervals of change of deformations and density instead of inequations (4), (5) and (7), (8) are defined from the following inequations under such circumstances:

$$(\varepsilon_0)_* < \varepsilon_0 < 0; \; \varepsilon_0^* < (\varepsilon_0)_*; \quad (11)$$

$$1 \le \frac{\rho}{\rho_0} \le \left(\frac{\rho}{\rho_0}\right)_*; \; \left(\frac{\rho}{\rho_0}\right)_* \le \left(\frac{\rho}{\rho_0}\right)^*; \quad (12)$$

$$\left(\frac{\rho}{\rho_0}\right)_* = \left[1+2(\varepsilon_0)_*\right]^{-\frac{3}{2}}; \; (\varepsilon_0)_* = \frac{1}{2}\left[(\lambda_1)_*^2 - 1\right].$$

*Table*

**Critical values $\varepsilon_0$ and $\frac{\rho}{\rho_0}$**

| $\nu$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| $(\lambda_1)_*$ | $\frac{0.78}{0.89}$ | $\frac{0.82}{0.91}$ | $\frac{0.84}{0.94}$ | $\frac{0.89}{0.95}$ | $\frac{0.94}{0.98}$ | $\frac{1}{1}$ |
| $\lambda_1^*$ | $\frac{0.50}{0.71}$ | $\frac{0.58}{0.76}$ | $\frac{0.67}{0.82}$ | $\frac{0.77}{0.87}$ | $\frac{0.87}{0.94}$ | $\frac{1}{1}$ |
| $(\varepsilon_0)_*$ | $\frac{-0.1958}{-0.1040}$ | $\frac{-0.1638}{-0.0860}$ | $\frac{-0.1472}{-0.0582}$ | $\frac{-0.1040}{-0.0488}$ | $\frac{-0.0582}{-0.0198}$ | $\frac{0}{0}$ |
| $\varepsilon_0^*$ | $\frac{-0.375}{-0.2480}$ | $\frac{-0.3318}{-0.2112}$ | $\frac{-0.2756}{-0.1638}$ | $\frac{-0.2036}{-0.1216}$ | $\frac{-0.1216}{-0.0582}$ | $\frac{0}{0}$ |
| $\left(\frac{\rho}{\rho_0}\right)_*$ | $\frac{2.1073}{1.4185}$ | $\frac{1.8137}{1.3270}$ | $\frac{1.6872}{1.2040}$ | $\frac{1.4185}{1.1664}$ | $\frac{1.2040}{1.0625}$ | $\frac{1}{1}$ |
| $\left(\frac{\rho}{\rho_0}\right)^*$ | $\frac{8}{2.7940}$ | $\frac{5.1253}{2.2780}$ | $\frac{3.3249}{1.8137}$ | $\frac{2.1904}{1.5186}$ | $\frac{1.5186}{1.2040}$ | $\frac{1}{1}$ |

Thus, changing of equilibrium state is implemented by loss of stability in case of influence of conservative forces on the cylindrical surface in bodies described by harmonic and quadratic elastic potentials. The body gets more stable curved geometric shape before the beginning of destruction process undergoing geometric forming in the local vicinity of the cavity. In its turn, this forming leads to unequal distribution of stress and deformations in the neighborhood of the cavity. This conclusion is not only related with solving the considered specific problem but also carries a general character.

**Numerical results and discussions**

In Table, along with the above mentioned discussion, numerical values of critical values of density corresponding to the "internal" instability and loss of stability of equilibrium state changes on geometric form. They show that the loss of stability of equilibrium state on geometric forming leads to "internal" instability i.e. to the beginning of destruction process for all values of Poisson's ratio in the considered case. Similar processes of local loss of stability of the equilibrium state occur in the neighborhood of the existing inclusions in the form of rods, bands, plates, etc. Therefore, the use of inequations (11)-(12) is more reasonable and correct in practice, especially in researches of the Earth's crust and upper mantle.

Such local deformation processes will influence on further change of density and other tectonophysical parameters in large geometric scales for geological time. In particular, processes of partial melting and phase transitions will not occur at single deep levels of the Earth's interior as in uniform deformation.

It's seen from the structure of formulae (2), (4)-(10) that, it's necessary to know only values of Poisson's ratio to define critical values of elongation (shortening) and strains.

Hofmeister [7] analyzed the problems of applicability of theoretical results to determine certainty of fields of Birch-Murnaghan's state equation (B-M EoS) received within Euler's tasking of finite deformation. Based on [12] experimental results, it is concluded that, theoretical model B-M EoS is unreliable for some solids as orthopyroxene. Let's consider this problem from the standpoint of the above received inequalities.

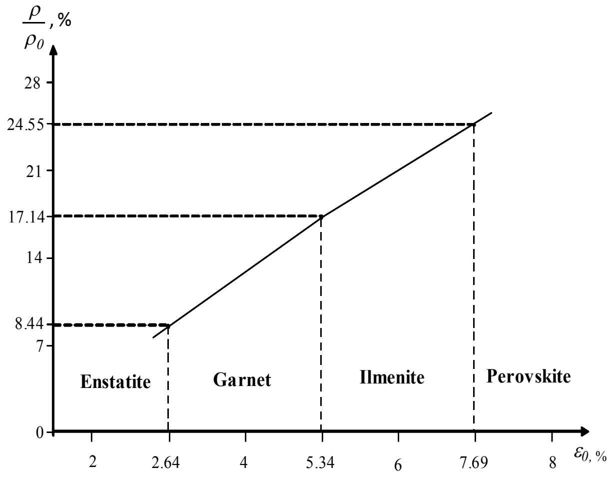


***Fig.*** Dependencies $\dfrac{\rho}{\rho_0}$ on $\varepsilon_0$ and sequence of phase transitions for orthopyroxene

It is assigned in the known Lin-Gun Liu's [11, 13] experiments that as a result of uniform deformation of sample from orthopyroxene (90% $MgSiO_3{\cdot}Al_2O_3$), phase transition from enstatite into garnet occurs in a relative decrease of the volume by 7,8% . The decrease in the volume by 8,0% causes a new phase transition from garnet into ilmenite. The further decrease in the volume by 6,9% leads to the phase transition from ilmenite into perovskite. Numerical calculations are conducted and their results are reflected in Figure using these experimental data.

Results show that enstatite sequently changes into garnet, ilmenite and into perovskite at last undergoing uniform deformation of pressure at its values 2,64%, 5,34% and 7,69%. An increase in the density of matter by 8,45%, 8,7% and 7,4% corresponds to these values of deformations. Comparison of these results with data in Table regarding $\varepsilon_0^*$ show that the sequence of phase transitions of orthopyroxene causes the destruction in all values of Poisson's ratio as a result of continuous deformation (in its modeling by harmonic potential). A similar conclusion is obtained for a quadratic potential except of the interval of changing the values of Poisson's ratio $\nu > 0.38$. Comparison of the results on the parameter $(\varepsilon_0)_*$ show that the local loss of stability of the elastic equilibrium state can cause separate phase transitions in the vicinity of inclusions in the form of the cylindrical cavity for a range of changes of Poisson's ratio $\nu > 0.38$ (harmonic potential) and $\nu \geq 0.12$ (quadratic potential). The obtained results cannot be considered reliable due to violation the term of uniformity of deformation process in such situation. It is known [10] that the value of Poisson's ratio averaged in Voigt-Reuss-Hill's approximation changes within $0{,}19 \leq \nu \leq 0{,}21$ in the interval of temperature $25^0 C \leq T \leq 700^0 C$ for orthopyroxene. Thus, results obtained within uniform deformations should be adjusted in case of presence the inhomogeneity as inclusion the form of the cylindrical cavity in orthopyroxene medium. The given conclusion relates to the case when deformation process is modeled by quadratic elastic potential. Furthermore, it follows from results in Figure that the parameters of physico-mechanical properties of orthopyroxene are undergone significant changes in the deformed state due to the realized phase transitions. Numerical values of parameters of physico-mechanical properties of these rocks differ among themselves significantly. This example shows quite clearly the difficulties in assessing the confidence intervals of state equations on criteria of fundamental moduli of the elasticity. Apparently, the conclusions on the unreliability of the model B-M EoS are related with the mentioned circumstances here to describe experimental data of orthopyroxene.

## Conclusion

Properly involving the theory of finite deformations to problems of determining different parameters of theoretical models of the Earth's development allow formulating reasonable deformation criteria on their confidence intervals. Apparently, these criteria are the most universal, simplest and comfortable to apply.

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## YERİN MODEL PARAMETRLƏRİNİN DAYANIQLIQ VƏ ETİBARLILIQ İNTERVALLARI

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Göstərilmişdir ki, deformasiyaların müntəzəm dəyişdiyi intervallar mövcuddur və bu intervallarda geoloji mühitin parametrlərinin təyini şərtləri saxlanılır. İntervalların sərhədləri, deformasiyaların müxtəlif dayanıqlığın itirilməsi proseslərinə uyğun olan böhran qiymətləri ilə təyin olunur.

***Açar sözlər:*** *Yerin nəzəri modeli, sonlu deformasiya, sıxlıq, tarazlıq vəziyyətinin dayanıqlığı*

## ИНТЕРВАЛЫ УСТОЙЧИВОСТИ И ДОСТОВЕРНОСТИ МОДЕЛЬНЫХ ПАРАМЕТРОВ ЗЕМЛИ

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Показано, что деформации имеют конкретные интервалы непрерывного изменения, в пределах которых сохраняются условия механики деформируемого твердого тела по определению параметров среды. Критические значения деформаций, соответствующих различным процессам потери устойчивости, определяют границы этих интервалов.

***Ключевые слова:*** *теоретические модели Земли, конечная деформация, плотность, устойчивость состояния равновесия*