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SOLUTION OF ONE PROBLEM FOR A PARABOLIC TYPE LINEAR LOADED DIFFERENTIAL EQUATION

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An application of the finite difference method to the solution of one problem for a parabolic type linear loaded differential equation with integral conditions is given in the paper. The finite difference method is applied to the solution of the problem. The corresponding difference problem is constructed, under certain conditions the convergence of the method is proved and the rate of convergence is determined.

Keywords: loaded differential equation, finite difference method, maximum principle

Statement of the problem

Problems for loaded differential equations occur in many areas of natural science (see, for example, [1], [2]). For example, the works [3] - [7] were devoted to the solutions of such problems.

The following problem for a linear loaded differential equation of parabolic type with a changeable coefficients is examined in this paper:

to find a continuous function $u = u(x, t)$ in a closed domain $\bar{D} = \{0 \leq x \leq l, 0 \leq t \leq T\}$, that satisfies the equation

$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(k(x) \frac{\partial u}{\partial x} \right) + bu(x, t) + \sum_{k=1}^m b_k u(x, \bar{t}_k) + f(x, t), 0 < x < l, 0 < t \leq T, \quad (1)$$

integral conditions

$$\begin{aligned} \int_0^l c_1(x) u(x, t) dx &= \mu_1(t), \\ 0 \leq t \leq T, \quad (2) \\ \int_0^l c_2(x) u(x, t) dx &= \mu_2(t), \end{aligned}$$

and initial condition

$$u(x, 0) = \varphi(x), \quad 0 \leq x \leq l. \quad (3)$$

Here

$$k(x) \geq k_0 > 0, f(x, t), \mu_1(t), \mu_2(t), c_1(x),$$

$c_2(x), \varphi(x)$ – are known continuous functions of their arguments, $b, b_k, k = 1, 2, \dots, m$ – real numbers, $\bar{t}_k, k = 1, 2, \dots, m$ – points of interval $(0, T]$.

Further, let's suppose that the problem (1)-(3) has a unique solution.

Replacement of integral conditions

Consider the first condition in (2) and differentiate it with respect to variable t :

$$\int_0^l c_1(x) \frac{\partial u(x, t)}{\partial t} dx = \mu_1'(t).$$

Hence, we have:

$$\begin{aligned} \int_0^l c_1(x) \left[\frac{\partial}{\partial x} \left(k(x) \frac{\partial u}{\partial x} \right) + bu(x, t) + \sum_{k=1}^m b_k u(x, \bar{t}_k) + \right. \\ \left. + f(x, t) \right] dx = \mu_1'(t). \quad (4) \end{aligned}$$

Applying the formula of integration by parts twice to the integral of the first term, we obtain:

$$\begin{aligned} \int_0^l c_1(x) \frac{\partial}{\partial x} \left(k(x) \frac{\partial u}{\partial x} \right) dx &= c_1(x) k(x) \frac{\partial u(x, t)}{\partial x} \Big|_{x=0}^{x=l} - \\ &- c_1'(x) k(x) u(x, t) \Big|_{x=0}^{x=l} + \int_0^l C_1(x) u(x, t) dx, \end{aligned}$$

where $C_1(x) = (c_1'(x) k(x))'$.

Finally, applying the trapezoid formula to the last integral on the right side, we obtain the validity of equality

$$\int_0^l c_1(x) \frac{\partial}{\partial x} \left(k(x) \frac{\partial u}{\partial x} \right) dx = c_1(x) k(x) \frac{\partial u(x, t)}{\partial x} \Big|_{x=0}^{x=l} - c_1'(x) k(x) u(x, t) \Big|_{x=0}^{x=l} + h \left[\frac{C_1(x_0)}{2} u(x_0, t) + C_1(x_1) u(x_1, t) + \dots + C_1(x_{N-1}) u(x_{N-1}, t) + \frac{C_1(x_N)}{2} u(x_N, t) \right] + O(h^2).$$

Taking into account this equality in (4), after elementary transformations, we come to the following equality:

$$c_1(x) k(x) \frac{\partial u(x, t)}{\partial x} \Big|_{x=0}^{x=l} - c_1'(x) k(x) u(x, t) \Big|_{x=0}^{x=l} + h \cdot \left[\frac{C_1(x_0)}{2} u(x_0, t) + C_1(x_1) u(x_1, t) + \dots + C_1(x_{N-1}) \cdot u(x_{N-1}, t) + \frac{C_1(x_N)}{2} u(x_N, t) \right] = \mu_1'(t) - b\mu_1(t) - \sum_{k=1}^m b_k \mu_1(\bar{t}_k) - \int_0^l c_1(x) f(x, t) + O(h^2). \quad (5)$$

Similarly, we have:

$$c_2(x) k(x) \frac{\partial u(x, t)}{\partial x} \Big|_{x=0}^{x=l} - c_2'(x) k(x) u(x, t) \Big|_{x=0}^{x=l} + h \cdot \left[\frac{C_2(x_0)}{2} u(x_0, t) + C_2(x_1) u(x_1, t) + \dots + C_2(x_{N-1}) u(x_{N-1}, t) + \frac{C_2(x_N)}{2} u(x_N, t) \right] = \mu_2'(t) - b\mu_2(t) - \sum_{k=1}^m b_k \mu_2(\bar{t}_k) - \int_0^l c_2(x) f(x, t) + O(h^2), \quad (6)$$

here $C_2(x) = (c_2'(x) k(x))'$.

Thus, instead of integral conditions (2), we have conditions (5) and (6).

After excluding from conditions (5) and (6), first $\frac{\partial u(l, t)}{\partial x}$, then $\frac{\partial u(0, t)}{\partial x}$, instead of these conditions, under conditions (1) on the boundaries $x = 0$, $x = l$ and the domain $\bar{D}$, we'll obtain conditions of the form

$$k \left(\frac{h}{2} \right) \frac{u(x_1, t) - u(0, t)}{h} - \frac{h}{2} \left[\frac{\partial u(0, t)}{\partial t} - bu(0, t) - \sum_{k=1}^m b_k u(0, \bar{t}_k) - f(0, t) \right] - \frac{c_1'(0) c_2(l) - c_1(l) c_2'(0)}{c_1(0) c_2(l) - c_1(l) c_2(0)} \cdot k(0) u(0, t) + \frac{c_1'(l) c_2(l) - c_1(l) c_2'(l)}{c_1(0) c_2(l) - c_1(l) c_2(0)} k(l) u(l, t) + h \left[\frac{c_1(l) C_2(x_0) - C_1(x_0) c_2(l)}{2(c_1(0) c_2(l) - c_1(l) c_2(0))} u(x_0, t) + \frac{c_1(l) C_2(x_1) - C_1(x_1) c_2(l)}{c_1(0) c_2(l) - c_1(l) c_2(0)} u(x_1, t) + \dots + \frac{c_1(l) C_2(x_{N-1}) - C_1(x_{N-1}) c_2(l)}{c_1(0) c_2(l) - c_1(l) c_2(0)} u(x_{N-1}, t) + \frac{c_1(l) C_2(x_N) - C_1(x_N) c_2(l)}{2(c_1(0) c_2(l) - c_1(l) c_2(0))} u(x_N, t) \right] = \bar{\mu}_1(t) + O(h^2), \quad (7)$$

$$k \left(l - \frac{h}{2} \right) \frac{u(x_N, t) - u(x_{N-1}, t)}{h} + \frac{h}{2} \left[\frac{\partial u(l, t)}{\partial t} - bu(l, t) - \sum_{k=1}^m b_k u(l, \bar{t}_k) - f(l, t) \right] - \frac{c_1'(0) c_2(0) - c_1(0) c_2'(0)}{c_1(0) c_2(l) - c_1(l) c_2(0)} k(0) u(0, t) + \frac{c_1'(l) c_2(0) - c_1(0) c_2'(l)}{c_1(0) c_2(l) - c_1(l) c_2(0)} k(l) u(l, t) + h \left[\frac{c_1(0) C_2(x_0) - C_1(x_0) c_2(0)}{2(c_1(0) c_2(l) - c_1(l) c_2(0))} u(x_0, t) + \dots \right]$$

$$\begin{aligned}
& + \frac{c_1(0)C_2(x_1) - C_1(x_1)c_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} u(x_1, t) + \dots + \\
& + \frac{c_1(0)C_2(x_{N-1}) - C_1(x_{N-1})c_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} u(x_{N-1}, t) + \\
& + \frac{c_1(0)C_2(x_N) - C_1(x_N)c_2(0)}{2(c_1(0)c_2(l) - c_1(l)c_2(0))} u(x_N, t) \Big] = \\
& = \bar{\mu}_2(t) + O(h^2). \quad (8)
\end{aligned}$$

Here $\bar{\mu}_1(t)$ and $\bar{\mu}_2(t)$ – are known functions.

Difference problem

Divide the Ox -axis segment $[0, l]$ with dots $x_n = nh$, $n = 0, 1, 2, \dots, N$, $h = l/N$, by N equal parts, and Ot -axis segment $[0, T]$ with dots $t_j = j\tau$, $j = 0, 1, 2, \dots, j_0$, $\tau = T/j_0$, by j_0 equal parts. We choose the step τ so that the points $\bar{t}_k$, $k = 1, 2, \dots, m$ are among the points $t_j = j\tau$, $j = 1, 2, \dots, j_0$. Suppose, that $\bar{t}_k = t_{j_k}$, $k = 1, 2, \dots, m$, $t_{j_1} < t_{j_2} < \dots < t_{j_m}$. Define in the domain $\bar{D}$ a grid $\bar{\omega}_{h\tau} = \{(x_n, t_j), n = 0, 1, 2, \dots, N, j = 0, 1, 2, \dots, j_0\}$.

Using, instead of integral conditions (2), the last equalities (7) and (8), problem (1) - (3) can be associated the following difference problem on the grid $\bar{\omega}_{h\tau}$:

$$\begin{aligned}
& \frac{h}{2} \frac{y_0^{j+1} - y_0^j}{\tau} - \frac{1}{2} k \left(\frac{h}{2} \right) \left(\frac{y_1^{j+1} - y_0^{j+1}}{h} + \frac{y_1^j - y_0^j}{h} \right) - \\
& - \frac{h}{2} \left(b \frac{y_0^{j+1} + y_0^j}{2} + \sum_{k=1}^m b_k y_0^{j_k} \right) + \\
& + \frac{c'_1(0)c_2(l) - c_1(l)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(0) \frac{y_0^{j+1} + y_0^j}{2} - \\
& - \frac{c'_1(l)c_2(l) - c_1(l)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(l) \frac{y_N^{j+1} + y_N^j}{2} -
\end{aligned}$$

$$\begin{aligned}
& - h \left[\frac{c_1(l)C_2(x_0) - C_1(x_0)c_2(l)}{2(c_1(0)c_2(l) - c_1(l)c_2(0))} \frac{y_0^{j+1} + y_0^j}{2} + \right. \\
& + \frac{c_1(l)C_2(x_1) - C_1(x_1)c_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \frac{y_1^{j+1} + y_1^j}{2} + \dots + \\
& + \frac{c_1(l)C_2(x_{N-1}) - C_1(x_{N-1})c_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \frac{y_{N-1}^{j+1} + y_{N-1}^j}{2} + \\
& + \left. \frac{c_1(l)C_2(x_N) - C_1(x_N)c_2(l)}{2(c_1(0)c_2(l) - c_1(l)c_2(0))} \frac{y_N^{j+1} + y_N^j}{2} \right] = f_0^j, \\
& \frac{y_n^{j+1} - y_n^j}{\tau} - \frac{1}{2} k \left(x_n + \frac{h}{2} \right) \frac{y_{n+1}^{j+1} - y_n^{j+1} + y_{n+1}^j - y_n^j}{h^2} + \\
& + \frac{1}{2} k \left(x_n - \frac{h}{2} \right) \frac{y_n^{j+1} - y_{n-1}^{j+1} + y_n^j - y_{n-1}^j}{h^2} - \\
& - b \frac{y_n^{j+1} + y_n^j}{2} - \sum_{k=1}^m b_k y_n^{j_k} = f_n^j, \\
& n = 1, 2, \dots, N-1, \quad (9)
\end{aligned}$$

$$\begin{aligned}
& \frac{h}{2} \frac{y_N^{j+1} - y_N^j}{\tau} + \frac{1}{2} k \left(l - \frac{h}{2} \right) \left(\frac{y_N^{j+1} - y_{N-1}^{j+1}}{h} + \frac{y_N^j - y_{N-1}^j}{h} \right) - \\
& - \frac{h}{2} \left(b \frac{y_N^{j+1} + y_N^j}{2} + \sum_{k=1}^m b_k y_N^{j_k} \right) - \\
& - \frac{c'_1(0)c_2(0) - c_1(0)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(0) \frac{y_0^{j+1} + y_0^j}{2} + \\
& + \frac{c'_1(l)c_2(0) - c_1(0)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(l) \frac{y_N^{j+1} + y_N^j}{2} + \\
& + h \left[\frac{c_1(0)C_2(x_0) - C_1(x_0)c_2(0)}{2(c_1(0)c_2(l) - c_1(l)c_2(0))} \frac{y_0^{j+1} + y_0^j}{2} + \right. \\
& + \frac{c_1(0)C_2(x_1) - C_1(x_1)c_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \frac{y_1^{j+1} + y_1^j}{2} + \dots + \\
& + \left. \frac{c_1(0)C_2(x_{N-1}) - C_1(x_{N-1})c_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \frac{y_{N-1}^{j+1} + y_{N-1}^j}{2} + \right.
\end{aligned}$$

$$+ \frac{c_1(0)C_2(x_N) - C_1(x_N)c_2(0)}{2(c_1(0)c_2(l) - c_1(l)c_2(0))} \frac{y_N^{j+1} + y_N^j}{2} \Big] = f_N^j,$$

$$j = 0, 1, \dots, j_0 - 1,$$

$$y_n^0 = \varphi(x_n), \quad n = 0, 1, \dots, N. \quad (10)$$

Here

$$f_0^j = -\bar{\mu}_1(t_j + 0,5\tau) + \frac{h}{2} f(0, t_j + 0,5\tau),$$

$$f_N^j = \bar{\mu}_2(t_j + 0,5\tau) + \frac{h}{2} f(l, t_j + 0,5\tau),$$

$$f_n^j = f(x_n, t_j + 0,5\tau), \quad n = 1, 2, \dots, N - 1.$$

It should be noted that the difference problem (9) - (10) approximates problem (1) - (3) with an accuracy of $O(h^2 + \tau^2)$, if the solution $u(x, t)$ of problem (1) - (3) has bounded partial derivatives with respect to variable x up to the fourth, with respect to variable t up to the third orders on the domain $\bar{D}$ and equation (1) is satisfied both on the boundaries $x = 0$ and $x = l$ of the domain $\bar{D}$.

The solution of difference problems of the form (9) - (10) is presented, for example, in the author's paper [7]. Therefore, we will not stop on the solution of this difference problem.

Maximum principle and consequences

Theorem 1 (Maximum principle). Suppose, that grid function $y_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$, satisfies problem (9) - (10). Suppose, that conditions $f_n^j \leq 0$ ($f_n^j \geq 0$), $n = 0, 1, \dots, N, j = 0, 1, \dots, j_0 - 1$ are satisfied. If the following conditions are satisfied

$$0 < k_0 \leq k(x) \leq k_1, \quad b_k > 0, \quad k = 1, 2, \dots, m,$$

$$b + \sum_{k=1}^m b_k \leq 0, \quad \frac{c'_1(l)c_2(l) - c_1(l)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \geq 0,$$

$$\frac{c'_1(0)c_2(0) - c_1(0)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \geq 0,$$

$$\frac{c'_1(0)c_2(l) - c_1(l)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(0) -$$

$$- \frac{c'_1(l)c_2(l) - c_1(l)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(l) \geq \varepsilon > 0,$$

$$\frac{c'_1(l)c_2(0) - c_1(0)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(l) -$$

$$- \frac{c'_1(0)c_2(0) - c_1(0)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} k(0) \geq \delta > 0, \quad (11)$$

$$\tau \leq \min \left\{ \frac{2h^2}{2k_1 - bh^2}, h^2 \left[k_0 + \frac{bh^2}{2} + k_0 + \frac{bh^2}{2} + hk(0) \frac{c'_1(0)c_2(l) - c_1(l)c'_2(0)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \right]^{-1}, \right. \\ \left. h^2 \left[k_0 - \frac{bh^2}{2} + hk(l) \frac{c'_1(l)c_2(0) - c_1(0)c'_2(l)}{c_1(0)c_2(l) - c_1(l)c_2(0)} \right]^{-1} \right\},$$

then solution $y_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$ of the problem (9)-(10), differ from constant, cannot take the greatest positive (smallest negative) value at $n = 0, 1, \dots, N, j = 1, 2, \dots, j_0$.

Theorem 2. Suppose, that grid function $y_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$, satisfies to the problem (9)-(10).

If $f_n^j \leq 0, \varphi(x_n) \leq 0$ ($f_n^j \geq 0, \varphi(x_n) \geq 0$), $n = 0, 1, \dots, N, j = 0, 1, \dots, j_0 - 1$ and conditions (11) are satisfied, then $y_n^j \leq 0$ ($y_n^j \geq 0$), $n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$.

The validity of the statement of this theorem follows from the maximum principle.

Consequence. Suppose, that conditions (11) are satisfied. Then the homogeneous problem corresponding to problem (9) - (10) has only the trivial solution

$$y_n^j = 0, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0.$$

It follows from this consequence that under conditions (11) there is a unique solution to the difference problem (9) - (10).

Theorem 3 (Comparison theorem).

Suppose, that $y_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$ - solution of the difference problem (9)-(10), and $\tilde{y}_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$ - solution of the difference problem, obtained by replacing in (9) - (10) the functions $f_n^j, n = 0, 1, \dots, N, j = 0, 1, 2, \dots, j_0 - 1$ and $\varphi(x_n), n = 0, 1, \dots, N$ by $\tilde{f}_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0 - 1$, and $\tilde{\varphi}(x_n), n = 0, 1, \dots, N$ correspondingly. Then, if $|f_n^j| \leq \tilde{f}_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0 - 1$ and $|\varphi(x_n)| \leq \tilde{\varphi}(x_n), n = 0, 1, \dots, N$ then under conditions (11) inequalities $|y_n^j| \leq \tilde{y}_n^j, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$ are takes places.

Convergence

Let's define the grid function z_n^j in the grid domain $\bar{\omega}_{h\tau}$ by the equality

$$z_n^j = y_n^j - u(x_n, t_j), n = 0, 1, \dots, N, j = 0, 1, \dots, j_0,$$

where y_n^j is the solution of the difference problem (9) - (10), $u(x_n, t_j)$ is the value of the exact solution of problem (1) - (3) at the point (x_n, t_j) of the grid $\bar{\omega}_{h\tau}$.

After substituting the expression y_n^j found from the last equality in the difference problem (9) - (10), can be obtained a difference problem of the form (9) - (10) with respect to function z_n^j , only with the difference that, in the right-hand sides of these equations, instead of f_n^j there will be approximation er-

rors $\psi_n^j, n = 1, 2, \dots, N - 1, j = 0, 1, \dots, j_0$, correspondingly, and the right-hand side of the initial condition will be equal to zero.

Using the comparison theorem, for solution on the difference problem with respect to z_n^j the following was proved.

Theorem 4. Suppose, that solution of equation (1) in domain $D = \{0 < x < l, 0 < t \leq T\}$ have bounded partial derivatives up to the fourth order with respect to variable x , up to the third order with respect to the variable t and this equation is satisfied both on the boundaries $x = 0$ and $x = l$ of the domain $\bar{D}$. If conditions (11) and the condition $b + \sum_{k=1}^m b_k \leq -B < 0$ are satisfied, then the solution of the difference problem (9) - (10) converges to the solution of the problem (1) - (3). In this case the following estimate takes place $|y_n^j - u(x_n, t_j)| \leq L\xi(h^2 + \tau^2) \cdot 2l, n = 0, 1, \dots, N, j = 0, 1, \dots, j_0$.

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PARABOLİK TİP XƏTTİ YÜKLƏNMİŞ DİFERENSİAL TƏNLİK ÜÇÜN BİR MƏSƏLƏNİN HƏLLİ

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İşdə parabolik tip dəyişən əmsallı xətti yüklənmiş diferensial tənlik üçün integral şərtlə bir məsələ tədqiq edilmişdir. Integral şərtlər qeyri-lokal sərhəd şərtləriylə əvəz olunduqdan sonra, qurulmuş yeni məsələnin həllinə sonlu fərqlər üsulu tətbiq edilmiş, məsələni ikinci tərtib dəqiqliklə approksimasiya edən fərq məsələsi qurulmuşdur. Müəyyən şərtlər daxilində fərq məsələsinin həllinin ilkin məsələnin həllinə yığılması isbat olunmuş və yığılma sürəti təyin edilmişdir.

Açar sözlər: yüklənmiş diferensial tənlik, sonlu fərqlər üsulu, maksimum prinsipi

РЕШЕНИЕ ОДНОЙ ЗАДАЧИ ДЛЯ ЛИНЕЙНОГО НАГРУЖЕННОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ПАРАБОЛИЧЕСКОГО ТИПА

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В работе исследована одна задача для линейного нагруженного дифференциального уравнения параболического типа с интегральными условиями. После замены интегральных условий с нелокальными граничными условиями к решению полученной новой задачи применен метод конечных разностей и построена разностная задача, аппроксимирующая ее со вторым порядком точности. При выполнении некоторых условий доказана сходимость разностной задачи и получена оценка для скорости сходимости.

Ключевые слова: нагруженное дифференциальное уравнение, метод конечных разностей, принцип максимума