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MHD WAVES AND INSTABILITIES IN THE COLLISIONLESS SPACE PLASMA

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Based on the 16-moment MHD transport equations, the propagation of linear waves in an anisotropic homogeneous cosmic plasma is considered. A general dispersion relation is derived with allowance for two plasma components (electrons and protons) and heat flux along the magnetic field. This dispersion relation is a generalization of the previously studied cases of one-component (ion) plasma. The case in which the effects associated with the heat flux are ignored is analyzed in more detail. In the limit of longitudinal propagation,the wave modes fully consistent with the modes known in the low-frequency kinetic theory of collisionless plasma are classified. Firehose and mirror instabilities are analyzed. It is shown that taking into account the electron component modifies the growth rates and thresholds of instabilities.

1. INTRODUCTION

Since the measured parameters of highly rarefied magnetized space plasmas (e.g., solar and stellar winds, star coronas, star disks, the ionosphere and magnetosphere of planets, and interstellar medium) are macroscopic, the MHD description of such plasmas is preferable. The derivation of a closed set of MHD equations for collisionless plasma runs into difficulties. The main difficulty is related to the truncation of the infinite chains of equations for the moments of the distribution functions. This requires additional physical justification, as well as a specific type of the particle velocity distribution. Classical examples of such equations describing plasma as a fluid are the Chew–Goldberger–Low (CGL) equations [1] and the 16-moment transport equations [2, 3], derived for a bi-Maxwellian plasma with a zero Larmor radius. The main advantage of the 16-moment MHD transport

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equations in comparison with the CGL equations is that they take into account the heat flux along the magnetic field. In contrast to the CGL equations, the 16-moment equations give a correct expression for the threshold of mirror instability, which is identical to the result predicted by the lowfrequency kinetic theory [4,5]. A disadvantage of the MHD descriptions of plasma in comparison with the kinetic one is that it considers small wave numbers $k < \omega_{pp}/c$ (where ω_{pp} is the proton plasma frequency and c is the speed of light). In a number of papers, MHD instabilities were modified with allowance for a finite Larmor radius (see, e.g., [6,7]).

In the previous works, we developed the theory of MHD instabilities based on the 16-moment equations [4,5,8,9]. In those works, the results were obtained for an ion plasma. The role of electrons was reduced only to the maintenance of plasma quasineutrality. Strictly speaking, the contributions of the plasma electron component can be ignored under the condition $T_e \ll T_p$, which is rarely satisfied in reality. Here, we generalize the theory of linear MHD instabilities with allowance for the electron component and its anisotropy and study the thresholds for the onset of firehose and mirror instabilities in an electron–proton anisotropic plasma.

2. MHD TRANSPORT EQUATIONS IN ANISOTROPIC PLASMA

The kinetic description of the dynamic phenomena in a plasma consisting of electrons and ions is based on the Boltzmann–Vlasov evolutionary equations for the distribution functions $f_\alpha(\mathbf{u};\mathbf{r};t)$ of each particle species $\alpha = \{e,i\}$. If it is necessary to take into account the effect of the electromagnetic field, these equations are complemented with Maxwell's equations. The macroscopic plasma parameters of interest to us (the density, macroscopic flow velocity, pressure, and heat flux) are determined as integral moments of the distribution functions in the three-dimensional space of microscopic velocities $\mathbf{u}$. In the moving frame of reference, these moments are represented as

$$n = \int f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u}, \quad n\,\mathbf{v} = \int \mathbf{u}f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u},$$

$$p = m\int |\mathbf{u}-\mathbf{v}|^2 f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u},$$

$$p_{\|} = m\int [(\mathbf{u}-\mathbf{v})\cdot\mathbf{b}]^2 f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u},$$

$$S_{\|} = (m/2)\int [(\mathbf{u}-\mathbf{v})\cdot\mathbf{b}]^3 f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u},$$

$$S_B = (m/2)\int [(\mathbf{u}-\mathbf{v})\cdot\mathbf{b}]|\mathbf{u}-\mathbf{v}|^3 f(\mathbf{u};\mathbf{r};t)d^3\mathbf{u},$$

where $\mathbf{b}$ is a unit vector along the magnetic field, the mean total pressure $p = (2p_\perp + p_\|)/3$ is determined by the transverse ($p_\perp$) and longitudinal ($p_\|$)

pressures, and the total longitudinal heat flux $S_B = S_\perp + S_\parallel$ is defined as the sum of the longitudinal heat fluxes caused by the transverse ($S_\perp$) and longitudinal ($S_\parallel$) thermal motions. The number of these and more high rank integral moments can be arbitrarily large, and they are all expressed via one another. The chain of equations describing these moments (transport equations) can also be infinite. Additional physically justified conditions for truncating the chain of equations are needed. In the case of a dense collisional plasma in which the equilibrium particle distribution functions are close to Maxwellian, these chains of equations are truncated easily. This results in the usual MHD equations for an isotropic plasma. However, in the case of rare collisions and in the presence of a strong magnetic field, the particle distribution functions are not Maxwellian and the truncation of the chain of the moment equations for a nonequilibrium plasma is problematic. In this case, the solution of the kinetic equation for each particle species is usually sought as an expansion about a given distribution function with an anisotropic temperature with respect to the direction of the external magnetic field. If this function is assumed to be bi-Maxwellian (the simplest form for an anisotropic plasma), then, for very small Larmor radii of particles gyrating in the magnetic field ($r_B \to 0$) , a system of 16-moment equations is obtained [2, 3]. In the generally accepted notation, these equations are written as

$$\frac{d\rho_\alpha}{dt} + \rho_\alpha \mathtt{div}\mathbf{v} = 0, \tag{1}$$

$$\rho\frac{d\mathbf{v}}{dt} + \frac{1}{4\pi}\mathbf{B}\times(\nabla\times\mathbf{B}) + \sum_\alpha \left[\nabla p_{\alpha\perp} + (\mathbf{B}\cdot\nabla)\left(\frac{p_{\alpha\parallel} - p_{\alpha\perp}}{B^2}\mathbf{B}\right)\right] = 0, \tag{2}$$

$$Bp_{\alpha\parallel}\frac{d}{dt}\ln\left(\frac{B^2 p_{\alpha\parallel}}{n^3}\right) + \mathbf{B}\cdot\nabla S_{\alpha\parallel} + 2\left(S_{\alpha\perp} - \frac{1}{2}S_{\alpha\parallel}\right)\mathbf{B}\cdot\nabla\ln B = 0, \tag{3}$$

$$Bp_{\alpha\perp}\frac{d}{dt}\ln\left(\frac{p_{\alpha\perp}}{Bn}\right) + \mathbf{B}\cdot\nabla S_{\alpha\perp} - 2S_{\alpha\perp}\mathbf{B}\cdot\nabla\ln B = 0, \tag{4}$$

$$BS_{\alpha\parallel}\frac{d}{dt}\ln\left(\frac{B^3 S_{\alpha\parallel}}{2n^4}\right) + \frac{3p_{\alpha\parallel}}{m_\alpha}\mathbf{B}\cdot\nabla\left(\frac{p_{\alpha\parallel}}{n}\right) = 0, \tag{5}$$

$$BS_{\alpha\perp}\frac{d}{dt}\ln\left(\frac{S_{\alpha\perp}}{n^2}\right) + \frac{p_{\alpha\parallel}}{m_\alpha}\mathbf{B}\cdot\nabla\left(\frac{p_{\alpha\perp}}{n}\right) - \frac{(p_{\alpha\parallel} - p_{\alpha\perp})\,p_{\alpha\perp}}{nm_\alpha}\mathbf{B}\cdot\nabla\ln B = 0, \tag{6}$$

$$\frac{d\mathbf{B}}{dt} + \mathbf{B}\mathtt{div}\mathbf{v} - (\mathbf{B}\cdot\nabla)\,\mathbf{v} = 0, \quad \mathtt{div}\mathbf{B} = 0. \tag{7}$$

When deriving these equations, it is assumed that the plasma is quasineutral, $n_e \approx n_i = n$, and the mass velocities of its components are close, $\mathbf{v}_e \approx \mathbf{v}_i = \mathbf{v}$. Here, $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla, \rho_\alpha = nm_\alpha$, and $S_{\alpha\parallel}$ and $S_{\alpha\perp}$ are the field-aligned heat fluxes caused by the longitudinal and transverse thermal motions of the particles of species α. If we neglect these fluxes, $S_{\alpha\parallel} = 0$ and $S_{\alpha\perp} = 0$, we obtain the laws of variation in the longitudinal and transverse thermal energies along the trajectory of a fluid element (the left-hand sides of Eqs.(3) and (4)). This pair of equations (the so-called double adiabats) and Eqs.(1), (2), and (7) form a closed system of equations known as the CGL equations [1]. However, if we use the CGL equations, Eqs.(5) and (6) remain unsatisfied, because, when deriving the CGL equations, the third moments of the distribution function were omitted without any justification; i.e., the heat fluxes were not taken into account. Equations (1)–(7) given here contain heat fluxes and form a more complete system. The CGL equations do not follow from these equations as a special case.

3. DISPERSION RELATION OF THE WAVES

For simplicity, we consider the case in which plasma in the unperturbed state is uniform and stationary: all quantities $\mathbf{v}_0, \rho_0, p_{\parallel 0}, p_{\perp 0}, \mathbf{B}_0, S_{\parallel 0}$, and $S_{\perp 0}$ for particles of species α do not depend on the coordinates and time. Equations (1)–(7) automatically satisfy these conditions with nonzero heat fluxes. Let us consider small perturbations of physical quantities with respect to the equilibrium state. For example, we represent the pressure as $p = p_0 + p'(r,t)$, where $p'(r,t) \sim \exp i(\mathbf{k}\cdot\mathbf{r} - \omega_0 t)$ and $|p'| \ll p_0$. Here, $\omega_0 = \omega + (\mathbf{v}_0 \cdot \mathbf{k})$ is the oscillation frequency in the frame of reference moving with the plasma and $\mathbf{k}$ is the wave vector of oscillations. For the small wave perturbations we can derive dispersion relations. Uncompressible wave modes are separated from the common system and their dispersion relation is:

$$\omega^2 = c_A^2 k_\parallel^2 \left(1 - 4\pi \sum_\alpha \frac{p_{\alpha\parallel} - p_{\alpha\perp}}{B^2}\right). \tag{8}$$

This is a prototype of the dispersion relation for Alfven wave modes in an isotropic plasma. In the dimensionless parameteres this may written as $\omega^2/c_{i\parallel}^2 k_\parallel^2 = \beta_i + \varphi_i - 1 + (\varphi_e - 1)/\Psi^2$, where $\beta_i = B^2/(4\pi p_{i\parallel}), \varphi_\alpha = T_{\alpha\perp}/T_{\alpha\parallel}, \Psi^2 = T_{i\parallel}/T_{e\parallel}$, c_i - ion sonic velocity. Under the condition $\beta_i + \varphi_i + \varphi_e/\psi^2 < 1 + 1/\psi^2$, Alfven modes become unstable and firehose instability arises. In two cases, the growth rate of firehose instability passes to the well-known case: for $\psi^2 \gg 1$ (cold electrons, $T_{e\parallel} \ll T_{i\parallel}$) and for isotropic electrons, $\varphi_e = 1$. If the plasma electron component is anisotropic,

$\varphi_e \neq 1$, then, for $\varphi_e > 1$ ($T_{e\perp} > T_{e\|}$), firehose instability is suppressed, whereas in the opposite case, $\varphi_e < 1$ ($T_{e\perp} < T_{e\|}$), on the contrary, instability intensifies.

Other dispersion relation is a 12th-degree polynomial equation for the normalized phase velocity $x = \omega/k_{\|}c_{i\|}$,

$$U_{12}x^{12} + U_{10}x^{10} + U_8x^8 + U_6x^6 + U_4x^4 + U_2x^2 + U_0 + \\ +\gamma_\alpha\left[U_9x^9 + U_7x^7 + U_5x^5 + U_3x^3 + U_1x\right] = 0, \quad (9)$$

which is a general dispersion relation for compressible wave modes in an infinite homogeneous anisotropic magnetized two-component plasma. Here, we took into account the field-aligned heat fluxes carried by particles of species α. The coefficients U_{0-12} of the equation are complicated real functions of the parameters of the problem. These coefficients are given in the Appendix of the paper [10]. Dispersion relations of the waves (slow ion-acoustic (SIA), slow electron-acoustic (SEA), fast magnetosonic (FMS), slow sound (SS), fast ion-acoustic (FIA) and fast electron-acoustic (FEA) modes) in the fluxless case are shown in the fig.1 and fig.2: For a different set of parameters, when condition for the onset of fire-

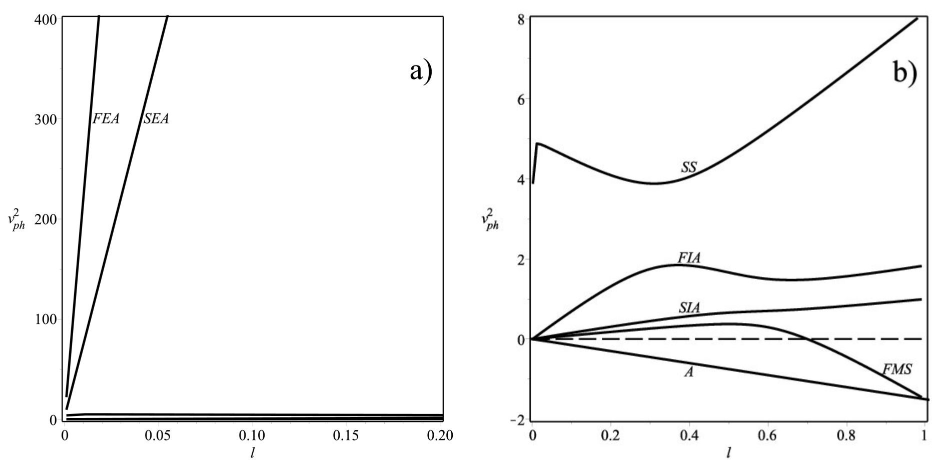

Fig. 1. Wave phase velocity squared vs. propagation angle parameter $l = \cos^2\theta$ in the case arising of firehose instability: $\varphi_e = \varphi_i = 0.5, \beta_i = 1$, and $\psi = 0.5$. Instability arises if $\nu^2_{ph} < 0$. (a) Electronic acoustic waves and (b) all other wave branches. Only the Alfven (A) and fast magnetosonic (FMS) modes become unstable.

hose instability is not satisfied, mirror instability may arise at large propagation angles. Such an example is presented in Fig.2, where mirror instability develops on the branch of the FIA mode. For mirror instability, there is also a threshold.

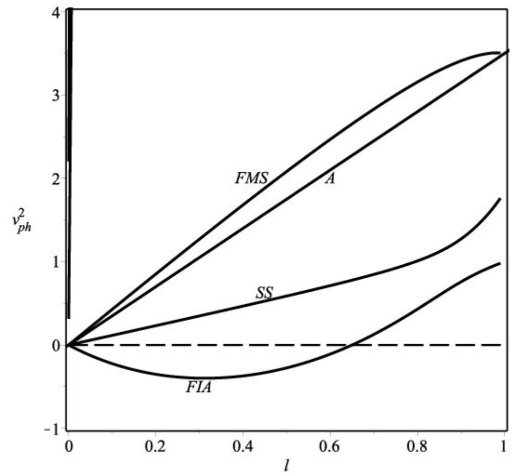

Fig. 2. Wave phase velocity squared vs. propagation angle parameter $l = \cos^2\theta$ in the case where firehose instability condition is not satisfied: $\varphi_e = \varphi_i = 1.5, \beta_i = 1$, and $\psi = 0.5$. In this case, mirror instability of quasi-transverse fast ion-acoustic (FIA) modes is possible.

An increase in the magnetic field ($\beta_i \sim B^2$) suppresses the instabilities under consideration. This can be seen from the examples presented in Fig.3.

4. CONCLUSIONS

In the previous theoretical studies of MHD instabilities in anisotropic plasmas, mainly in the CGL and 16-moment approximations, the role of electrons was ignored. It was reduced only to ensuring plasma quasineutrality. However, under actual space conditions, both ion and electron plasma components are substantially nonisothermal ($T_e \neq T_i$) and anisotropic ($T_\perp \neq T_\parallel$). Our main goal was to clarify the effect of the presence of the electron component on the conditions for the onset of the known types of MHD instabilities in an anisotropic plasma. To this end, we used 16-moment MHD transport equations with allowance for the heat flux in a multicomponent bi-Maxwellian plasma. It is shown that allowance for electrons introduces into the problem new parameters associated with the plasma nonisothermality, the anisotropy of the electron component, and the electron heat flux. For simplicity, with neglect of heat fluxes, the role of the electron component in the onset of firehose and mirror instabilities has been studied in detail. It is shown that, in the actually observed parameter ranges, the electron component cannot be ignored. The criteria for the onset of instabilities, as well

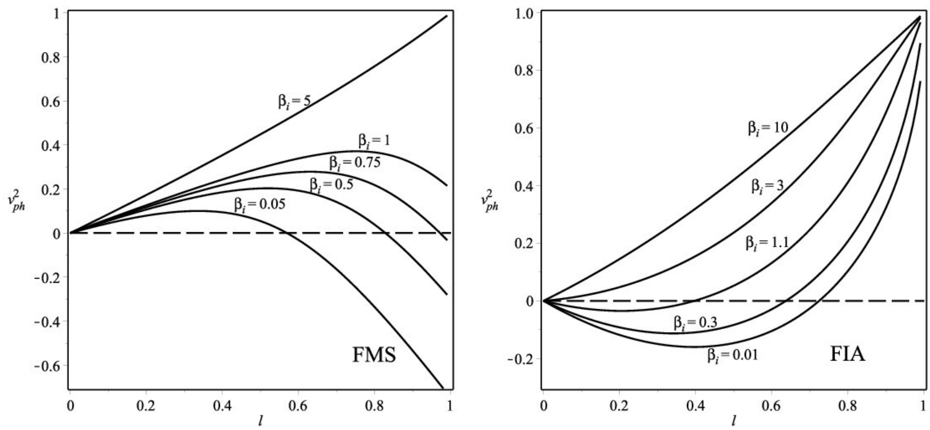

Fig. 3. Effect of the magnetic field (parameter β_i) on the condition for the onset of instability: (a) phase velocity squared vs. propagation angle of the FMS modes (the second firehose instability) for $\varphi_e = 0.5, \varphi_i = 0.7$, and $\psi = 1$; (b) FIA modes (mirror instability) for $\varphi_e = 1.2, \varphi_i = 1.5$, and $\psi = 0.7$.

as their growth rates, depend substantially on the parameters $\psi^2 = T_{i\|}/T_{e\|}$ and $\varphi_e = T_{e\perp}/T_{e\|}$.

In the absence of field-aligned heat fluxes, there are three types of MHD instabilities: incompressible parallel firehose instability, compressible oblique firehose instability, and compressible mirror instability. All the instabilities are aperiodic; i.e., the real part of the frequency is zero, Re(ω)=0. This is primarily a consequence of neglecting the Landau damping in deriving the MHD equations. The inclusion of dissipative effects (e.g., heat fluxes, Hall effects, etc.) will stabilize the instability, and the instability will become oscillatory. The main disadvantage of the obtained expressions for the growth rates of MHD instabilities is that these growth rates are linear functions of the wavenumber, Im(ω)$\sim k$. This means that, on very small scales ($k \to \infty$), the instability growth rates increase without bound. The reason is that the 16-moment MHD transport equations used are derived under the assumption of a zero Larmor radius.

The properties of firehose and mirror instabilities are well-known from the low-frequency kinetic theory [13–16]. The influence of a finite Larmor radius on the thresholds and growth rates of kinetic instabilities, as well as their stabilization, is widely discussed in the literature (see, e.g., [6, 17, 18]). For wavelengths on the order of the ion Larmor radius, the effective elasticity of the magnetic field lines increases substantially, which leads to a maximum growth rate and an increase in the threshold for mirror instability [17]. At shorter wavelengths, the effective

electric field acting on the ions decreases (due to the averaging caused by Larmor gyration). This leads to a decrease in the growth rate toward shorter wavelengths. In the cases under consideration, the electrons were mainly assumed to be isotropic and cold. In a more general case, where the electrons of a bi-Maxwellian plasma are anisotropic and not cold, the conditions for the onset of mirror instability are substantially modified [15, 19–23]. It is found that the maximum growth rate of mirror instability is smaller when the electrons are isotropic; however, if anisotropy appears, the growth rate increases. The finite Larmor radius effects in the presence of anisotropic electrons were considered in [7, 24]. It was found that suppression of instability by the effects related to the finite Larmor radii of electrons and ions depends substantially on the degree of anisotropy of the electron temperature. The influence of the finite Larmor radius effects on firehose instability has been studied by many authors (see, e.g., [6, 25–30]). The main result is that instability is suppressed at small spatial scales. It is very difficult to take into account the finite Larmor radius and dissipative effects in a fluid model of magnetized collisionless plasma. In the simplest case in which field-aligned heat fluxes are disregarded and double adiabats (CGL equations) are satisfied, an attempt to describe firehose modes by using such an approach was made in [31]. In that work, short-wavelength firehose instability was stabilized by including the Hall damping and finite ion Larmor radius effects. Analysis of instability of a collisionless magnetized plasma in the fluid model in a more general case in which heat fluxes, finite Larmor radius effects [32], and weak collisions between particles [33] are taken into account is a very complicated but important problem.
The results of this work can be used to interpret the observed low-frequency large-scale turbulence in the solar and stellar wind plasma. Note that more details about the results presented here can be found in our published work [34].

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