

# ON PARTICULAR SOLUTIONS OF MHD EQUATIONS FOR A SINGLE-FLUID COLLISIONLESS PLASMA OF ANISOTROPIC SOLAR WIND

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In this work the MHD theory of the solar wind is considered, taking into account the anisotropy of the heat flux and the thermal pressure of the plasma. The resulting equations have singularities. The dependence of the solar wind speed on the radial distance is complex and depends on the thermal velocities of plasma particles. In this study, special solutions for different comparable cases of the components of the solar wind speed and the thermal velocity of the plasma are considered. As a result, the expressions depending on the speed of the solar wind for different distances from the sun and the radial distance for the Mach number were obtained. Found formulas are particular solutions. The obtained expressions play an important role in determining of the speed of the solar wind on the whole heliosphere. The radial dependences of the solar wind velocity and thermal velocities are investigated for several special cases.

**Keywords:** Magnetohydrodynamics (MHD)–sun–solar wind (SW)–corona–heliosphere

## 1. INTRODUCTION

The solar wind (SW) as a physical phenomenon that is not only of academic interest, with the study of processes in plasma, which is in the natural conditions of outer space, but is also a factor that must be taken into account when studying the processes occurring in the vicinity of the Earth and affecting our a life. Solar wind streams strongly affect the Earth's magnetosphere, its structure, and non-stationary processes on the Sun can lead to magnetic storms, disrupting radio communications, also affects weather-sensitive people. The solar wind parameters strongly depend on latitude, plasma anisotropy and on the solar magnetic field. The solar magnetic field is the main cause of anisotropy in SW plasma.

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CW is highly variable and a wide variety of variations in its physical properties are observed in a wide range of amplitudes, spatial and temporal scales. The presence of these variations shows that the expansion of the corona is a much more complex process than the stationary outflow of a uniform plasma flow, as Parker originally assumed. The characteristic time scales of the phenomena in this case vary over a wide range from fractions of a second (plasma waves and "noises") to tens of hours and days (variations associated with the inhomogeneity of the solar corona and large-scale changes in its structure).

First the fluid description of SW in a collisionless plasma was undertaken with certain assumptions in the framework of the CGL MHD approximation, which has known limitations as when studying shock waves in this approximation, heat fluxes were not taken into account, which can significantly change the properties of linear and shock waves [1–4]. The radial and stationary outflow of plasma from the Sun was simulated on the basis of usual isotropic MHD equations. It was recognized that the complete disregard for parallel heat fluxes for a collisionless plasma is not entirely justified [5]. In real situations during plasma flow from the sun – there appears an anisotropy in the solar wind rare collisional plasma, and the heat fluxes along the magnetic field in the solar wind become important.

A system of equations for a collisionless plasma, including heat fluxes along the magnetic field, was proposed in [5, 6]. Heat fluxes are the result of an asymmetric distribution function of particles in the plasma, which, according to measurements, more realistically reflects the conditions in the SW [7]. When studying the radial spherical expansion of a stationary SW, the problem is reduced to solving a system of three nonlinear ordinary differential equations. Early some particular solutions of these MHD equations for one-liquid fluid collisionless plasma of the anisotropic solar wind have been found, which can be used to find global solutions [8–10].

## BASIC EQUATIONS

For the radial plasma propagation  $B_\phi = 0$  and  $V_\phi = 0$ , and if we take into account that all the parameters of the solar wind depend only on the radial distances  $r$ , then  $B = B_r$ ,  $h_r = 1$ ,  $h_\phi = 0$ , and after solving the basic equations and assuming the longitudinal propagation of the plasma in the direction magnetic field, which is chosen perpendicular to the surface of the Sun, six integral constants and three differential equations [7] were obtained for spherical coordinates,

$$r^2 B = C_1, r^2 \rho V = C_2 \quad (1)$$

$$\frac{V^2}{2} + \frac{1}{\rho} \left( P_\perp + \frac{3}{2} P_\parallel \right) + \frac{1}{\rho V} \left( S_\perp + \frac{1}{2} S_\parallel \right) - \frac{GM}{r} = \frac{C_5}{C_2} \quad (2)$$

$$r^4 (p_{\perp} V + S_{\perp}) = C_1 C_6 \tag{3}$$

$$r^4 \rho^2 \frac{d}{dr} \left( \frac{V_r P_{\parallel}}{r^2 \rho^2} \right) + \frac{d}{dr} (r^2 S_{\parallel}) - 4r S_{\perp} = 0 \tag{4}$$

$$\frac{1}{V^2} \frac{d}{dr} \left( S_{\perp} \frac{V^3}{\rho} \right) + \frac{3}{2} \frac{d}{dr} \left( \frac{P_{\parallel}}{\rho} \right)^2 = 0 \tag{5}$$

$$\frac{d}{dr} \left( r^2 S_{\perp} \frac{V}{\rho} \right) + \frac{P}{\rho} \frac{d}{dr} \left( r^2 \frac{P_{\perp}}{\rho} \right) - 2r \left( \frac{P_{\perp}}{\rho} \right)^2 = 0 \tag{6}$$

From expressions (1-3), four constants are obtained and there are still three differential equations. Passing over the dimensionless quantities and parameters, where  $u_{\parallel}^2 = pV\rho$  and  $u_{\perp}^2 = p_{\perp}/\rho$  thermal velocities, we obtained [7–9]:

$$\left( \frac{Y}{X} - 1 \right) \frac{dX(x)}{dx} - 2 \frac{dY(x)}{dx} + \frac{4}{x^3} Z(x) - \frac{2\bar{g}}{x^2} = 0 \tag{7}$$

$$(\bar{C}_6 - Z(x)) \frac{dX(x)}{dx} + (Y(x) - X(x)) \frac{dZ(x)}{dx} - \frac{2}{x^3} Z^2(x) = 0 \tag{8}$$

$$\left( \frac{Y(x)}{X(x)} - 1 \right) \frac{dY(x)}{dx} + \frac{4}{3} \left( \bar{C}_5 + \frac{\bar{g}}{x} - \frac{\bar{C}_6}{x^2} - \frac{3}{4} X(x) - \frac{3}{2} Y(x) \right) \frac{1}{X(x)} \frac{dX(x)}{dx} + \frac{2}{3x^2} \left( \frac{2\bar{C}_6}{x} - \bar{g} \right) = 0 \tag{9}$$

where  $x = \frac{r}{R}$ ,  $X = X(x) = \frac{v^2}{v_0^2}$ ,  $Y = Y(x) = \frac{u_{\parallel}^2}{v_0^2}$ ,  $Z = Z(x) = x^2 \frac{u_{\perp}^2}{v_0^2}$ ,  $\bar{C}_5 = \frac{c_5}{C_2 v_0^2}$

$$\bar{C}_6 = \frac{C_1 C_6}{C_2 R v_0^2}, \quad \bar{g} = \frac{GM_0}{R v_0^2} \tag{10}$$

Here  $\nu_0$  is the characteristic speed for the solar wind (the speed of the solar wind near the Earth =  $V_E$ ): However, in the case of one-dimensional strictly parallel plasma motions, the equations for parallel and perpendicular mechanical energy become independent and can be written in a closed conservative form. In the spherical coordinate for the system, the equation has:

$$\rho U = \frac{C_2}{r^2}; S_{II} + 3P_{II}U + \rho U^3 = \frac{2K_1(r)}{r^2}; S_{\perp} + P_{\perp}U = \frac{C_6 C_2}{r^4}; B = \frac{C_1}{r^2} \tag{11}$$

where

$$K_1 = \frac{4}{3} \left( \bar{C}_5 + \frac{\bar{g}}{x} - \frac{\bar{C}_6}{x^3} \right), \quad K_2 = \frac{2}{3x^2} \left( 2\frac{\bar{C}_6}{x} - \bar{g} \right) \tag{12}$$

In the posed problem, the induction of the magnetic field of the sun depends on the distance (11). To study the singularity, we represent the equations in the form of differentials of each variable. And solving we get from (7) - (9)

$$3r^3 \frac{dX(r)}{dr} + \frac{f_1(r)}{f(r)} = 0, \quad 6r^3 \frac{dY(r)}{dr} + \frac{f_2(r)}{f(r)} = 0, \quad 6r^3 X(r) \frac{dZ(r)}{dr} + \frac{f_3(r)}{f(r)A(r)} = 0 \quad (13)$$

where

$$A(r) = \frac{Y(r)}{X(r)} - 1, B(r) = \frac{3}{4}(K1(r) - 3X(r)), D(r) = 2Z(r) - rg, E(r) = 2C_6 - rg \quad (14)$$

$$f(r) = \frac{A(r)}{2}(A(r) - 4) + \frac{4B(r)}{3X(r)}; f_1(r) = 3A(r)D(r) + 2E(r) \quad (15)$$

$$f_2(r) = A(r)f_1(r) - 6D(r)f(r) \quad f_3(r) = f_1(r)(D(r) - E(r)) - 12f(r)Z(r)^2 \quad (16)$$

**Approach to the solution:**The derived equations (16) are a generalization of the Parker problem to the case of anisotropic radial and stationary solar wind [7, p.10]. Found particular solutions for  $Y = 2X$  [8, p.339]. Several more particular solutions were found with the use of changes of variables [9, p.251, 10, p.19, 11, p.56]. The difficulty in solving these equations is associated with the zeros of the functions  $f(r)$  and  $A(r)$  in the domain of integration  $1 \leq r \leq \infty$ . But these equations contain two more unknown constants  $\bar{C}_5$  and  $\bar{C}_6$ , which make it possible to make the singularities removable. Let's study particular solutions. From (14)

$$Y(r) = (A(r) + 1)X(r) \quad (17)$$

When the condition

$$Z(r) \ll \frac{gr}{2} \text{ of (14)} \quad D(r) = 2Z(r) - rg \approx -gr$$

Of (8) considering (16) we get  $6r^3 X(r) \frac{dA(r)}{dr} - (2 + A(r)) \frac{f_1(r)}{f} - 6D(r) = 0$  of (13) and (14)

$$A(r) = (1/dX(r)) \left[ D_* - 2 \frac{dX(r)}{dr} \pm \sqrt{\left( D_* - 2 \frac{dX(r)}{dr} \right)^2 - \left( \frac{8B(r)}{3X(r)} \frac{dX(r)}{dr} + \frac{4}{3r^3} E(r) \right) \frac{dX(r)}{dr}} \right] \quad (18)$$

$$X(r)A(r) \frac{dZ(r)}{dr} + (C_6 - Z(r)) - \frac{2}{r^3} Z(r)^2 = 0$$

First case

1)  $Y(r) \ll X(r)$ ,  $A(r) \approx -1$ , at  $g = 1$  from (13)

$$\frac{1}{X(r)} \cdot \frac{dX(r)}{dr} + \frac{2}{3r^2} \cdot \frac{(4C_6 + r)}{\frac{8}{3}(C_5r^2 + r - C_6) - r^2X(r)} = 0 \tag{19}$$

And

$$\begin{aligned} \frac{d}{dr}(2Y(r) + X(r)) &\approx \frac{dX(r)}{dr} = -\frac{dX(r)}{dr} + \frac{6D(r)}{3r^3}, \quad dX(r) = \frac{dr}{r^2} \\ X(r) = \frac{1}{r} + C; \text{ at } r = \infty X(r) = 0, &\rightarrow C = 0, \text{ at } X(r) = X_\infty, C = X_\infty \end{aligned} \tag{20}$$

from (19)

$$\begin{aligned} AtC = 0, C_5 = C_6 = 0, X(r) = \frac{2}{r}, Y(r) = Z(r) = 0 \\ AtC \neq 0, C_5r^2 + r - C_6 = 0, r(C_6 + r) \left(\frac{1}{r} + C\right) = 0; \\ (1 + C_5)(r + C_6) = Cgr^2 + (g + C_6)r + C_6 \end{aligned}$$

$C_5 = C, g \equiv 1 + CC_6, -C_6 = C_6 = 0, C_5 = \frac{1}{2}X_\infty, Y(r) = Z(r) = 0, X(r) = \frac{2g}{r} + X_\infty,$   
 at  $X_\infty = 0, C_5 = C_6 = 0$ ; and ar  $Xt_\infty \neq 0, C_5 = \frac{1}{2}X_\infty, C_6 = 0$

Second case  $Y(r) = nX(r)n = const, A(r) = n - 1, n \geq 0.$

$$\begin{aligned} \frac{dA(r)}{dr} = 0, \quad X(r) \frac{dA(r)}{dr} + \frac{A(r) + 2}{2} \cdot \frac{dX(r)}{dr} - \frac{D(r)}{r^3} = 0, \\ \frac{dX(r)}{dr} = -\frac{2}{(n+1)r^2}; \quad X(r) = \frac{2}{(n+1)r} + 2C, \end{aligned}$$

There  $C_5r^2 + r - C_6 = q_2,$

And  $(n+1)(4C_6 - r(3n-1)) - 6r(\frac{1}{2}(n-1)(n-5) - 3) = q_1$

From (13) get  $\frac{8q_2}{3X(r)r} = q_1$

From here  $X(r) = \frac{8}{3r} \frac{q_2}{q_1} = \frac{2}{(n+1)r} + 2C, \frac{4}{3r} \frac{q_2}{q_1} = \frac{1}{(n+1)r} + C, \frac{4}{3} \frac{q_2}{q_1} - \frac{2}{n+1} - rC = 0;$   
 $\frac{4}{3}q_2 = q_1 \left(rC + \frac{1}{n+1}\right), \frac{4}{3}(C_5r^2 + gr - C_6) = \left(rC + \frac{1}{n+1}\right) \left(\frac{4C_6}{n_1} - grn_*\right)$

When  $n_* = \frac{3n-1}{n} + (n-1)(n-5), X(r) = \frac{2}{(n+1)r} + X_\infty, C_6 = 0, C_5 = X_\infty$

From (8) can find  $Z, (n-1)X(r) \frac{dZ(r)}{dr} + (C_6 - Z(r)) \frac{dX(r)}{dr} - \frac{2}{r^3}Z(r)^2 = 0$   
 considering  $X(r) = \frac{2}{(n+1)r} + X_\infty, C_6 = 0,$

$$(n-1) \left(\frac{2}{(n+1)r} + X_\infty\right) \frac{dZ(r)}{dr} + \frac{2}{(n+1)r^2}Z(r) - \frac{2}{r^3}Z(r)^2 = 0$$

Let's introduce a new variable. Let's admit

$$\tau = \frac{1}{r}, r \in [1, \infty], \tau \in [0, 1]; g_n = \frac{2}{(n+1)}, \quad \frac{dZ(r)}{dr} = \tau^2 \left( -\frac{dZ(\tau)}{d\tau} \right)$$

$$(n-1)(g_n\tau + X_\infty) \frac{dZ(\tau)}{d\tau} - 2\tau Z(r)^2 + g_n Z(r) = 0, Z = Z(\tau)$$

A) At  $n = 0; g_0 = 2, Z(0) = Z_\infty, (n-1)(x+C) \frac{dZ(x)}{dx} + 2xZ(x)^2 - Z(x) = 0$

$$Z(x) = \frac{s(x)}{M(x)}; \quad S(x) = n(nx + Cn - x - C)^{\frac{1}{n-1}}$$

$$M(x) = -2(nx + Cn - x - C)^{\frac{1}{n-1}} - n^2 C_1, \quad Z(\tau) = \frac{Z_\infty g_0^2 X_\infty}{M1(\tau)};$$

$$M1(\tau) = -2Z_\infty \tau X_\infty \ln(g_0\tau + X_\infty) - 2Z_\infty X_\infty^2 \ln(g_0\tau + X_\infty) + g_0^3 \tau + g_0^2 X_\infty + 2Z_\infty \tau X_\infty + 2Z_\infty \tau X_\infty \ln(X_\infty) + 2Z_\infty X_\infty^2 \ln(X_\infty)$$

B) At  $n = 2, g_2 = \frac{2}{3}, Z(\tau) = \frac{g_2\tau + X_\infty}{\tau^2 Z_\infty + X_\infty} \cdot Z_\infty;$  A at  $Z(1) = Z_0$  get

A) At  $n = 0, Z(\tau) = \frac{-Z_0 g_0 (g_0 + X_\infty)}{M2(\tau)}, M2(\tau) = 2Z_0 X_\infty - g_0^2 \tau - g_0 X_\infty - 2Z_0 g_0 \tau X_\infty$

B) At  $n = 2, Z(\tau) = \frac{Z_0 (\tau g_0 + X_\infty)}{\tau^2 Z_0 - Z_0 + g_2 + X_\infty}; 2Z \ll gr, 2Z\tau \ll 1$

$$Y(r) = 2X(r), Z(r) = 0, X(r) = \frac{2g}{3r} + X_\infty, \quad X_\infty = 0, C_5 = C_6 = 0$$

$$Y = nX, 2Z \ll r \text{ From } \frac{dY}{dr} = \frac{1}{2} \left( \left( \frac{Y}{X} - 1 \right) \frac{dX}{dr} - \frac{2}{r^2} \right) \tag{21}$$

$$Y = nX, 2Z \ll r$$

From  $\frac{dY}{dr} = \frac{1}{2} \left( \left( \frac{Y}{X} - 1 \right) \frac{dX}{dr} - \frac{2}{r^2} \right);$

$$\frac{dX}{dr} \left( n^2 + 2n - 1 + \frac{2K1(r)}{X} \right) = \frac{2}{3r^2} \left( (3n - 1) - \frac{4C_6}{r} \right)$$

and else  $n^2 + 2n - 1 = 0 \quad n = -1 \pm \sqrt{2},$  and

$$\frac{dX}{dr} \left( \frac{2K1(r)}{X} \right) = \frac{2}{3r^2} \left( (3n - 1) - \frac{4C_6}{r} \right) \text{ in there } \frac{dX}{X} = \frac{r(3n - 1) - 4C_6}{4r^2 (C_5 r^2 + r - C_6)} dr$$

$$X = \left( \frac{\sqrt{4C_5 C_6 + 1} + 2C_5 r + 1}{\sqrt{4C_5 C_6 + 1} - 2C_5 r - 1} \right)^{\frac{C_1}{2\sqrt{4C_5 C_6 + 1}}} \cdot \left( \frac{\sqrt{C_5 r^2 + r - C_6}}{r} \right)^{\frac{C_2}{C_6}} \cdot e^{\frac{C_2}{r}}$$

there  $C1 = 2C_5 + \frac{1}{C_6} - \frac{3nC_5}{2C_6} - \frac{3n}{4C_6^2} + \frac{C_5}{2C_6} + \frac{1}{4C_6^2}$ ,  $C2 = \frac{3n}{4C_6} - \frac{1}{4C_6} - 1$ ;

and at

$$r \rightarrow \infty X_\infty = (-1)^{\frac{C_1}{\sqrt[3]{4C_5C_6+1}}} C_5 \frac{C_2}{2C_6} \tag{22}$$

;

Third case  $2Z(r) \ll r, \tau = \frac{1}{r}. \frac{dA(\tau)}{d\tau} + \frac{1}{2X(\tau)} \cdot \frac{dX(\tau)}{d\tau} - \frac{1}{X(\tau)} \approx 0$ ,

In at  $X(\tau) \cdot \frac{dZ(\tau)}{d\tau} - 2\tau Z(\tau)^2 + (Z(\tau) - C_6) \cdot \frac{dX(\tau)}{d\tau} = 0$

Find X in form  $X(\tau) = a\tau^2 + b\tau + X_\infty$

A)  $a = 0, b = 2, C_6 = 0, C_5 = \frac{1}{2}X_\infty$ , B)  $a = \frac{45}{25} \frac{1}{X_\infty}, b = \frac{14}{5}, C_6 = -\frac{49}{100} \frac{1}{X_\infty}, C_5 = \frac{13}{28} X_\infty$ ;

And get for A

$$A(\tau) = -\frac{1}{2} \cdot \ln(a\tau^2 + b\tau + X_\infty) + \frac{4g}{\sqrt{4aX_\infty - b^2}} \cdot \operatorname{arctanh}\left(\frac{b+2a\tau}{\sqrt{4aX_\infty - b^2}}\right) + \frac{1}{C_A} \cdot \operatorname{arctanh}\left(\frac{b+2a\tau}{\sqrt{4aX_\infty - b^2}}\right)$$

$$A(0) = -1, C_A = \frac{1}{2} \ln(X_\infty) - \frac{2g}{\sqrt{4aX_\infty - b^2}} \cdot \operatorname{arctanh}\left(\frac{b+2a\tau}{\sqrt{4aX_\infty - b^2}}\right) - 1$$

$$-1 \leq A(\tau) \leq -0,5; \tau_* = 2\sqrt{5} \sqrt{\frac{X_\infty}{7\tau + 5X_\infty}}, h_1 = \frac{3 - \sqrt{2}}{\sqrt{2} - 1}, h_2 = \frac{4 - \sqrt{2}}{\sqrt{2} - 1}$$

$$Z(\tau) = \frac{1}{2500F(\tau_*) \cdot X_\infty} \cdot (2450C_z X_\infty Y(h_1, \tau_*) + 49J(h_2, \tau_*)) \tag{23}$$

$$F(\tau_*) = \frac{\sqrt{2} - 1}{50} \cdot J(h_2, \tau_*) + (\sqrt{2} - 1)C_z X_\infty Y(h_2, \tau_*) -$$

$$-\frac{\tau_*}{2} \cdot \left(C_z X_\infty Y(h_1, \tau_*) + \frac{1}{50} J(h_1, \tau_*)\right); Z(1) = Z_0 \rightarrow 0$$

Fourth case

$$2Z \ll r, 2\tau Z \ll 1, |A| \ll 1, Y \ll X$$

$$A(\tau_0) \approx 0, A \approx (\tau - \tau_0) \frac{dA}{d\tau}(\tau = \tau_0); A = (\tau - \tau_0) \frac{dA}{d\tau}(\tau = \tau_0) \Rightarrow$$

$$\frac{dX}{d\tau} = \frac{2(2C_6\tau - 1)X}{4(C_5 + \tau - C_6\tau^2) - 9X'}$$

$$\frac{dA}{d\tau} = \frac{1}{X} \cdot \left(\frac{dX}{d\tau} - 1\right), AX \frac{dZ}{d\tau} + 2\tau Z^2 + (C_6 - Z) \frac{dX}{d\tau} = 0$$

Let us find X in the form

$$X(\tau_0) = X_0, X_0 = a\tau_0^2 + b\tau_0 + c, \tau_0 = \frac{1}{2} \left( 1 \pm \sqrt{1 + 4C_6C_5 - 6C_6X_0} \right)$$

$$A(\tau) = \ln \left( \frac{C_5 + \tau - C_6\tau^2}{C_5 + \tau_0 - C_6\tau_0^2} \right) + \frac{3}{h} \left( \operatorname{arctanh} \left( \frac{1 - 2\tau C_6}{h} \right) - \operatorname{arctanh} \left( \frac{1 - 2\tau_0 C_6}{h} \right) \right)$$

$$\frac{dA}{d\tau} \text{ ( at } \tau = \tau_0) = -\frac{1}{2} \frac{1 + 4C_6\tau_0}{C_5 + \tau_0 - C_6\tau_0^2}, X_0 \frac{dA(\tau_0)}{d\tau} = \frac{dX(\tau_0)}{d\tau} - 1 = -\frac{1}{3} (1 + 4C_6\tau_0)$$

$$X_0(\tau - \tau_0) \cdot \frac{dA(\tau_0)}{d\tau} \cdot \frac{dZ}{d\tau} + 2\tau_0 Z^2 + (C_6 - Z) \cdot (2a\tau_0 + b) = 0$$

$$\operatorname{arctanh} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right)$$

$$A(\tau) = \ln \left( N(h) \frac{C_5 + \tau - C_6\tau^2}{C_5 + \tau_0 - C_6\tau_0^2} \right), N(h) = \left( \frac{h+1-2\tau C_6}{h-1+2\tau C_6} \cdot \frac{h-1+2\tau_0 C_6}{h+1-2\tau_0 C_6} \right)^{\frac{3}{2h}}$$

$$X_0(\tau - \tau_0) \cdot \frac{dA(\tau_0)}{d\tau} \cdot \frac{dZ}{d\tau} + 2\tau_0 Z^2 + (C_6 - Z) \cdot (2a\tau_0 + b) = 0$$

$$(\tau - \tau_0) \cdot \frac{dZ}{d\tau} + mZ^2 + nZ + p = 0$$

$$m = -\frac{6}{1 + 4\tau_0 C_6} \tau_0, n = 2 \frac{1 - 2\tau_0 C_6}{1 + 4\tau_0 C_6}, p = -2C_6 \cdot \frac{1 - 2\tau_0 C_6}{1 + 4\tau_0 C_6}$$

Find  $Z = -\frac{p_*}{2m} \cdot \left( \frac{n}{p_*} + \operatorname{tg} \left( \frac{p_*}{2} \ln(\tau - \tau_0) + \frac{p_*}{2} C_z \right) \right) \quad P_* = \sqrt{4pm - n^2}, p_*^2 > 0$  (24)

$$mZ_0^2 + nZ_0 + p = 0, Z(\tau_0) = Z_0 = \frac{1}{2m} \cdot \left( -n \pm \sqrt{n^2 - 4np} \right)$$

Fifth case at  $A \gg 1, Y \gg X$ , from(8)

$$\frac{dX}{d\tau} = \frac{2}{A} (2\tau Z - 1) \Rightarrow A \frac{dX}{d\tau} = 2(2\tau Z - 1)$$

$$X \frac{dA}{d\tau} + \frac{1}{2} A \frac{dX}{d\tau} = 1 - 2\tau Z \Rightarrow X \frac{dA}{d\tau} = 2(1 - 2\tau Z)$$

$$XA \frac{dZ}{d\tau} + 2\tau Z^2 + (C_6 - Z) \frac{dX}{d\tau} = 0 \Rightarrow X \cdot A =$$

$$\text{const} = Y - X$$

$$A = \frac{\text{const}}{X}, \frac{C}{X} \cdot \frac{dX}{d\tau} = 2(2\tau Z - 1)$$



$$X(0) = C_*, \ln(X) = \frac{2}{c} \cdot \int_0^\tau (2\tau Z - 1) d\tau + C_*, \quad X_* \cdot \exp\left(\frac{2}{c} \cdot \int_0^\tau (2\tau Z - 1) d\tau\right)$$

$$\frac{C}{X} \cdot \frac{dX}{d\tau} = 2(2\tau Z - 1) \Rightarrow \frac{dX}{d\tau} = \frac{2C_*}{C} (2\tau Z - 1) \exp\left(\frac{2}{C} \cdot \int_0^\tau (2\tau Z - 1) d\tau\right) \text{ and}$$

$$C \frac{dZ}{d\tau} + 2\tau Z^2 + (C_6 - Z) (2\tau Z - 1) \frac{2C_*}{C} (2\tau Z - 1) \exp\left(\frac{2}{C} \cdot \int_0^\tau (2\tau Z - 1) d\tau\right) = 0,$$

$$2\tau Z \ll 1, X = C_* \cdot \exp\left(-\frac{2\tau}{C\tau}\right), A = \frac{C}{C_*} \exp\left(\frac{2\tau}{C}\right) \gg 1;$$

$$C \frac{dZ}{d\tau} + 2\tau Z^2 - \frac{2C_*}{C} \exp\left(-\frac{2\tau}{C}\right)_{Z_0c} = 0; \frac{C_*}{C} \gg 1; X(0) = C_* \rightarrow 0$$

, or  $C \rightarrow 0$ .

$$C \frac{dZ}{d\tau} + 2\tau Z^2 \approx 0, \quad Z(\tau_*) = Z_0, Z(\tau) = \frac{Z_0 C}{(\tau^2 - \tau_*^2) z_0 + C}; 2\tau Z \ll 1 \Rightarrow Z_0 \ll 1, \tau_* \rightarrow 0 \tag{25}$$

Particular cases can be continued, but with these solutions it is possible to characterize the dependence of the solar wind speed on the distance from the Sun. **Conclusions- results:** The MHD theory of anisotropic solar wind is considered, taking into account the heat flux and the difference in the thermal pressure components. The resulting equations have singularities. Found formulas in the work are particular solutions. The dependence of the solar wind speed on the radial distance is complex and depends on the thermal velocities of plasma particles. The obtained solutions can be used to construct a global solution to the solar wind equations numerically.

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