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SEARCH FOR CHAINS OF RESONANCES IN THE COMPACT PLANETARY SYSTEM KEPLER-51

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We consider the compact three-planetary system Kepler-51 and search for resonance chains within the errors of determining the orbit periods from observations.To determine the resonant combinations of the semi-major axes of the orbits, the ratio of the mean motions of neighboring planets was represented as a segment of a sequence of convergent fractions.We have shown that the planetary system Kepler-51 evolves outside the low-order resonant chains.

Keywords: Exoplanetary system – Resonances – Dynamic evolution – Stability

1. INTRODUCTION

At present, several dozen compact planetary systems containing more than two planets with masses of the order of the Earth's mass are known[1)].It is shown that the stable evolution of compact planetary systems requires the presence of resonances that prevent close encounters of planets moving in neighboring orbits (see, for example, the five-planet systems Kepler-80 [1], K2-138 [2] and the seven-planet system TRAPPIST-1 [3]).

We consider the compact three-planetary system Kepler-51 and search for resonance chains within the errors of determining the periods from observations. Antoniadou and Voyatzis [4] demonstrate three possible scenarios safeguarding Kepler-51, each followed by constraints. Firstly, there are the $2:1$ and $3:2$ two-body mean-motion resonances (MMRs), in which eccentricity $e_b < 0.02$, such that these two-body MMRs last for extended time spans. Secondly, there is the $1:2:3$ three-body Laplace-like resonance, in which $e_c < 0.016$ and $e_d < 0.006$ are necessary for such a chain to be viable.Thirdly, there is the combination comprising the $1:1$ secondary resonance inside the $2:1$ MMR for the inner pair of

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[1)] http://exoplanet.eu/catalog/

planets and an apsidal difference oscillation for the outer pair of planets in which the observational eccentricities, e_b and e_c, are favored as long as $e_d \approx 0$.

For compact systems, an important factor affecting evolution is tidal interaction; therefore, when analyzing the feasibility of the proposed resonance chains, we will model dynamic evolution taking into account tides. We introduce the resonant angles in Section 2. We will describe the technique for searching for resonant chains in Section 3. We will search for resonant chains in the Kepler-51 system in Section 4.

2. RESONANT ANGLES

When analyzing the resonant properties of planetary systems, the behavior of resonant angles is studied. For two planets i and $i+1$, which are in resonance of mean motions $k_i : (k_i - l_i)$, the resonant angle can be set as follows [5]:

$$\varphi_{i,\, i+1,\, i+s} = (k_i - l_i)\lambda_i - k_i\lambda_{i+1} + l_i\varpi_{i+s}, \tag{1}$$

where l_i is the resonance order, λ_i, λ_{i+1} are the mean longitudes of planets i and $i+1$, ϖ_{i+s} is the longitude of the periapsis of planet i ($s = 0$) or $i+1$ ($s = 1$) orbit. For the next pair of planets $i+1$ and $i+2$, which is in resonance $k_{i+1} : (k_{i+1} - l_{i+1})$, the resonant angle

$$\varphi_{i+1,\, i+2,\, i+1+s} = (k_{i+1} - l_{i+1})\lambda_{i+1} - k_{i+1}\lambda_{i+2} + l_{i+1}\varpi_{i+1+s}. \tag{2}$$

Instead of two resonances between two neighboring pairs of planets, three-body resonances can be considered. In this case, the three-body resonance is a chain of two two-body resonances $k_i : (k_i - l_i)$ and $k_{i+1} : (k_{i+1} - l_{i+1})$ with a resonant angle that does not depend on the periapsis longitude [5]:

$$\Phi^{p,\, p+q,\, q}_{i,\, i+1,\, i+2} = p\lambda_i - (p+q)\lambda_{i+1} + q\lambda_{i+2}, \tag{3}$$

where $p = l_{i+1}(k_i - l_i)$, $q = l_i k_{i+1}$.

3. SEARCH FOR POSSIBLE CHAINS OF RESONANCES

The search for chains of resonances was carried out for the values of the semi-major axes of the orbits of the planets, which varied within the standard deviation $a_i = a_{0i} + \sigma_{ai}$. Here a_{0i} is the nominal value of the semi-major axis of the orbit, σ_{ai} is the standard deviation of the determination of the semi-major axis. To determine the resonant combinations of the semi-major axes of the orbits, the ratio of the mean motions n_i and n_{i+1} was represented as a segment of a sequence of convergent fractions

$$\frac{n_i}{n_{i+1}} = \left\{\frac{b_1}{d_1}, \frac{b_2}{d_2}, \ldots, \frac{b_j}{d_j}\right\}, \qquad b_j < b_{max}. \tag{4}$$

where b_{max} is the maximum value of the numerator. We obtain a rational approximation of the real ratio of mean motions

$$\frac{n_i}{n_{i+1}} = \frac{k_i}{k_i - l_i} = \frac{b_j}{d_j}. \tag{5}$$

After the sets of relations of the form (5) are obtained for all pairs of neighboring planets, possible chains of resonances are formed (if the resonance values of the semi-major axes of the outer and inner orbits in neighboring pairs coincide). The final selection of potential resonance chains is based on an estimate of the frequency of the resonant angle

$$\nu_{i,\, i+1,\, i+2}^{p,\, p+q,\, q} = |pn_i - (p+q)n_{i+1} + qn_{i+2}| < \varepsilon, \tag{6}$$

where the values are chosen as a criterion $\varepsilon \sim 10^{-4} - 10^{-5}$ day^{-1}.

The algorithm showed high efficiency in searching for resonance chains in the compact planetary system K2-72 [6].

4. POSSIBLE RESONANT CHAINS IN A COMPACT PLANETARY SYSTEM KEPLER-51

Table 1 shows the parameters of the Kepler-51 system. Star mass m and radius R are given in solar mass $M_\odot$ and radius $R_\odot$, correspondently. The masses m_{pl} and radii R_{pl} of the planets are expressed in Earth's mass $M_\oplus$ and radius $R_\oplus$, respectively. The orbital elements T, e, and g are the period, eccentricity, and periapsis argument, correspondently. The moment T_{conj} corresponds to the conjunction of the planet with the star.

We used the data to search for resonant chains in the Kepler-51 system. We concluded that the system does not contain resonances of the 1$^{\text{st}}$, 2$^{\text{nd}}$, and 3$^{\text{rd}}$ orders, for resonant numerators not exceeding $b_{max} = 10$. We did not find confirmation of the realization of a resonant chain $2:1 - 3:2$ in a compact three-planetary system Kepler-51. For compact systems, an important factor affecting evolution is tidal interaction; therefore, we considered a number of scenarios for the evolution of the Kepler-51 system over 100 Myr using the Posidonius software [8], which takes into account tidal interactions. We concluded that the system can have a stable evolution over long-time in the absence of low-order resonances.

Table 1. Parameters of the compact three-planetary system Kepler-51 [7]

Parameter	Kepler-51	Kepler-51b	Kepler-51c	Kepler-51d
m $[M_\odot]$, m_{pl} $[M_\oplus]$	$0.894^{+0.036}_{-0.048}$	$2.48^{+1.23}_{-1.04}$	$3.14^{+0.50}_{-0.48}$	$5.22^{+1.17}_{-1.07}$
R $[R_\odot]$, R_{pl} $[R_\oplus]$	$0.841^{+0.023}_{-0.021}$	$6.62^{+0.19}_{-0.17}$	$8.98^{+2.84}_{-2.84}$	$9.04^{+0.25}_{-0.23}$
T [day]		$45.15393^{+0.00036}_{-0.00038}$	$85.31553^{+0.00138}_{-0.00109}$	$130.1827^{+0.0009}_{-0.0009}$
$e\cos g$		$-0.019^{+0.009}_{-0.010}$	$0.024^{+0.014}_{-0.014}$	$0.014^{+0.011}_{-0.011}$
$e\sin g$		$-0.059^{+0.019}_{-0.022}$	$-0.048^{+0.027}_{-0.029}$	$-0.037^{+0.022}_{-0.024}$
T_{conj}		$-1285.4040^{+0.0006}_{-0.0006}$	$-1274.4886^{+0.0030}_{-0.0030}$	$-1304.0693^{+0.0009}_{-0.0009}$

5. DISCUSSION AND CONCLUSIONS

We implement a method for searching for chains of resonances within the limits of errors in determining the values of the periods of planetary orbits. We concluded that the compact tree-planetary system Kepler-51 evolves outside the low-order resonant chains. We modeled the dynamic evolution of the Kepler-51 system using the Posidonius software, taking into account tides. We showed that the system can have a stable evolution in the absence of low-order resonances. In the future, we plan to refine the initial configurations of orbits and planets that ensure the stable evolution of the planetary system over long-time.

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