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COSMOLOGICAL VACUUM AND COSMOLOGICAL-QUANTUM EFFECTS

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The notion of cosmological quantum effect is introduced. These effects depend on the value of the cosmological constant. It is shown that dark matter and energy can be examples of cosmological quantum effects. It is suggested that the irreversibility of time is a cosmological quantum effect.

Keywords: cosmological quantum effect, de Sitter group $SO(4,1)$, dark matter, dark energy, cosmological constant, cosmological vacuum

1. INTRODUCTION

Usually, quantum effects are understood as microscopic and macroscopic quantum effects. As well-known microscopic quantum effects, one can indicate the photoelectric effect or the Compton effect, and as macroscopic quantum effects, the phenomena of superconductivity and superfluidity.

The phenomena of dark matter and energy are topical problems of fundamental science, [1–5]. In the world's leading laboratories, experimenters are trying to find the composition of dark matter from the point of view of elementary particles. Among the known elementary particles, sterile neutrinos are the most probable. Nevertheless, the search for hypothetical particles is also being actively pursued.

In our work, we tried to prove that the phenomena of dark matter and energy are not related to the composition of elementary particles, but are a consequence of a non-zero value of the cosmological constant. It is shown that quantum principles in cosmology can explain the phenomena of dark mass and energy, i.e. we are dealing with cosmological quantum effects.

Fundamental research at the intersection of high energy physics and cosmology is a trend in modern physics.

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2. DE SITTER COSMOLOGICAL MODELS

According to Einstein's general relativity, the metric properties of space-time are determined by the distribution and motion of matter, [6]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\lambda}_{\lambda} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}; \tag{1}$$

where

$R_{\mu\nu}$ – Ricci tensor, $g_{\mu\nu}$ – metric tensor, Λ – cosmological constant, G – the gravitational constant of Newton, $T_{\mu\nu}$ – energy-momentum tensor and $\lambda, \mu, \nu = 0, 1, 2, 3$.[1)]

This equation can be rewritten in an equivalent form:

$$R_{\mu\nu} = \Lambda g_{\mu\nu} - \frac{8\pi G}{c^4} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\lambda}_{\lambda}); \tag{2}$$

It is clear from (2) that the Λ-term, even in the absence of matter ($T_{\mu\nu} = 0$), changes the space-time geometry and $g_{kl}\Lambda$ is the energy-momentum tensor of the vacuum.

In the vacuum, the Einstein equations take the form:

$$R_{\mu\nu} = \Lambda g_{\mu\nu}. \tag{3}$$

For $\Lambda = 0$, the solution of (3) is the Minkowski manifold with a group of Poincaré motions.

The histories of the cosmological constant are reviewed in [7]- [8].

In the general case, the solutions of the Einstein (1)-(2) equations do not have a maximal group of motions. But in 1917 Willem de Sitter found two solutions of (3) for $\Lambda \neq 0$ with different global groups of movements, [9–11]:

$$ds^2 = \frac{dr^2}{1 - r^2/R^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) - \left(1 - \frac{r^2}{R^2}\right) c^2 dt^2, \qquad if\ \Lambda > 0; \tag{4}$$

$$ds^2 = \frac{dr^2}{1 + r^2/R^2} + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) - \left(1 + \frac{r^2}{R^2}\right) c^2 dt^2, \qquad if\ \Lambda < 0. \tag{5}$$

Here the radius of space R and the cosmological constant Λ are related by the following formula:[2)]

$$\Lambda = \pm\frac{3}{R^2} \tag{6}$$

[1)] The attentive reader will notice that, unlike the modern notation, the momentum tensor energy on the right side of the equation appears with a plus sign. The sign depends on the definition of Riemann-Christophel connections and the Riemann tensor. We have chosen to retain the classical form of Einstein's equations since this is not essential for our purposes.

[2)] It is easy to see that for $\Lambda \to 0$, i.e. $R \to \infty$ both solutions (4)-(5) are transferred to the flat world of Minkowski.

Using stereographic projections:

$$\xi_1 = r\cos\varphi; \quad \xi_2 = r\sin\vartheta\cos\varphi; \quad \xi_3 = r\sin\vartheta\sin\varphi;$$
$$\xi_4 = R\sqrt{1-\frac{r^2}{R^2}}\cosh\left(\frac{ct}{R}\right); \quad \xi_0 = R\sqrt{1-\frac{r^2}{R^2}}\sinh\left(\frac{ct}{R}\right)$$

and

$$\eta_1 = r\cos\varphi; \quad \eta_2 = r\sin\vartheta\cos\varphi; \quad \eta_3 = r\sin\vartheta\sin\varphi;$$
$$\eta_4 = R\sqrt{1+\frac{r^2}{R^2}}\cos\left(\frac{ct}{R}\right); \quad \eta_5 = R\sqrt{1+\frac{r^2}{R^2}}\sin\left(\frac{ct}{R}\right)$$

de Sitter solutions can be isometrically embedded as sub-manifolds in 5-dimensional pseudo-Euclidean spaces:

$$\xi_0^2 - \xi_1^2 - \xi_2^2 - \xi_3^2 - \xi_4^2 = -R^2 \tag{7}$$

and

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 - \eta_5^2 = -R^2, \tag{8}$$

respectively.

These spaces have global symmetry groups $SO(4,1)$ and $SO(3,2)$ that leave the metrics (7)-(8) invariant. Groups $SO(4,1)$ and $SO(3,2)$ are called de Sitter groups. Spaces (4), (7) and (5), (8) are called de Sitter worlds of the 1st and 2nd kind or according to the modern terminology, de Sitter worlds dS and anti–de Sitter AdS, respectively.

3. WIGNER'S ELEMENTARY QUANTUM SYSTEMS

Wigner introduced the concept of an elementary system and showed that irreducible representations of the group of motions of the world correspond to such systems, i.e. the space of these representations has no invariant subspaces, [12][3)].

The vacuum solutions of Einstein's equations have a maximal group of motions - these are the Poincarè groups for the Minkowski world and the de Sitter group for the de Sitter worlds, depending on the value of the cosmological constant.

Wigner proved that in the world of Minkowski the mass (energy of rest) and spin of the elementary particle are:

[3)] It should be clarified that in the case of quantum mechanics we are dealing with projective representations of the Poincarè group $ISO(3,1)$ or vector representations of its universal covering group $SL(2,C)$.

- invariants of the Poincarè quantum-mechanical group $\overline{P_{+}^{\uparrow}}$, which uniquely characterize unitary irreducible representations[4)];
- the consequences of space-time symmetries, i.e. they can be obtained without using the equations of dynamics.

Invariants of the inhomogeneous Lorentz group, in other words, the Poincarè group $ISO(3,1)$ (Casimir operators) are constructed using translation generators P_μ and 4-dimensional rotations $M_{\mu\nu}$ as follows, [13, 14]:

$$m^2 = P^\mu P_\mu; \qquad w^2 = w^\mu w_\mu = m^2 s(s+1); \qquad w_\varrho = \frac{1}{2}\epsilon_{\lambda\mu\nu\varrho}P^\lambda M^{\mu\nu}, \tag{9}$$

where

m – mass, s – spin of the particle, and w_ϱ is the Pauli-Lubansky-Bargmann vector:

$$(m, s), \quad m^2 \geq 0, \qquad s = 0, 1/2, 1, 3/2, 2, ...$$

Important! In the case $m^2 > 0$, there is a third invariant - the sign of energy:

$$\varepsilon = \frac{P_0}{|P_0|} = \pm 1 \tag{10}$$

The application of the Wigner elementary system concept in the de Sitter world leads to interesting results. In de Sitter's world, elementary systems are also characterized by two parameters, [15]:

- The spin s retains its status;
- Instead of mass, a function of the rest energy of the special theory of relativity, spin, and radius of curvature of the Universe appears:

$$m^2 = m_0^2 + \frac{9}{4R^2} - \frac{s(s+1)}{R^2}. \tag{11}$$

  It is obvious that $m^2 \to m_0^2$ at $R \to \infty$.

  Here, the radius of curvature R is determined by a cosmological constant Λ, (6).

Passing to the limit $R \to \infty$ is an example of a contraction operation for Lie groups, their algebras, and representations in the sense of Wigner-Inönü, [16].

It can be seen from the formula that, in contrast to the Minkowski world, where the mass is a classical quantity in the de Sitter model, the second characteristic of the elementary system has a cosmological-quantum nature.

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4) $\overline{P_{+}^{\uparrow}}$ is isomorphic to $SL(2, C)$.

In this case the role of the contraction operation is similar to that of the Bohr correspondence principle, [17,19]. The operation of contraction replaces the principle of the correspondence of Bohr, since we do not have a complete theory of quantum gravity.

4. COSMOLOGICAL QUANTUM EFFECTS IN THE DE SITTER WORLD

Maximal groups of motions make it possible to use the well-known methods of relativistic quantum field theory in cosmology. Using the representations of the de Sitter group of motions, it is shown that the phenomena of dark matter and energy are a cosmological quantum effect. In de Sitter's world, gravity and anti-gravity are different states of Wigner's elementary quantum systems. But, in the world of Minkowski, anti-gravity can be excluded because there are no transitions between the two states. Our conclusion is based on the interpretation of the theorem on the contraction of representations of de Sitter group to representations of the quantum-mechanical Poincarè group, which we proved earlier, [20].

The result of the contraction of the UIR's, $T^{(\sigma,s)}(g)$, $g \in SO(4,1)$, $\sigma = -3/2+imR$, for $R \to \infty$ is the direct sum of UIR, $U^{(m,s;\,\varepsilon)}(g)$, $g \in ISO(3,1)$, with mass m, spin s and differing in energy sign ε.

In the de Sitter world, elementary Wigner systems are identified by spin and by a parameter, which is the flat limit of a function of spin and mass, with different energy signs. However, unlike the Minkowski world, we can not exclude negative energies from consideration. It is natural to identify states with opposite signs of the parameter ε as gravity and anti-gravity, which are characteristic of dark matter and energy.

But, in the general case, a solution to the problem of quantizing the gravitational field is required. This problem cannot be solved in the same way as an electromagnetic field or a general gauge field. The first main problem is that the general theory of relativity postulates a fundamental experimental fact about the equality of gravitational and inertial mass. According to the well-known work of Bohr-Rosenfeld [21], it is precisely due to a decrease in the ratio of the charge of a test particle to its mass that a consistent interpretation of quantum measurements in quantum electrodynamics is possible. And according to the principle of equivalence, this ratio is always equal to one and cannot be reduced.

Another problem is that all gravitating bodies, including test particles, have a gravitational or Schwarzschild radius.

And this makes the introduction of a local field problematic.

Given these and other difficulties in quantizing the gravitational field, it is usually said that the gravitational field is not quantized. But these problems of quantization of the gravitational field can be characterized differently: as a result of the unification of the general theory of relativity and quantum mechanics, a new physical object appears, different from the relativistic quantum field.

The next difficulty is related to the non-trivial symmetry properties of the vacuum solutions of Einstein's equations, which create serious difficulties in determining the quantum vacuum. And the principle of general covariance in general relativity makes the interpretation of solutions and the definition of the energy-momentum tensor ambiguous.

Without the maximal group of motions, using the methods of relativistic quantum field theory is impossible. Thus, the gravitational field cannot be quantized, and general relativity is not applicable in the microcosm of quantum particles. A reasonable question arises: is it possible to apply quantum mechanics on the scale of the Universe?

All this forces us to look for other possibilities for solving the problems of high-energy physics and cosmology.

5. ALGEBRA OF OBSERVABLES AND COSMOLOGY

The main reason is the difficulty of applying the principles of quantum mechanics to the Universe as a whole since the measuring device, which should be a classical system, is part of the world.

Historically, three formulations of quantum mechanics are known: Heisenberg's matrix mechanics, Schrodinger wave mechanics, and Jordan's observable algebra. The first two formulations of quantum mechanics are presented in monographs, where it is shown that the influence of the measuring device on the measurement process is manifested in the procedure of wave packet reduction, [**?**, 27]. J. Neumann focused on the fact that the Schrödinger equation only determines the evolution of the quantum system and does not describe the measurement procedure. The effect of a classical measuring device can be understood as a reduction of the wave function.

To find the correct application of quantum mechanics in cosmology, we have tried to find a representation of observable algebra that describes the interacting classical quantum system. To do this, we used the definition of the algebra of observables by Jordan in the formulation of [29, 30].

We have proved the theorem that an interacting classical and quantum system cannot be Hamiltonian system, [31].

There is not any representation of the observable algebra (1)-(4) corresponding to the interacting Hamilton classical-quantum system.

Bohr's Copenhagen interpretation is usually presented in books on quantum mechanics. But initially, Bohr proposed the idea of a fundamentally uncontrolled interaction between a classical system (measuring device) and a quantum system in quantum mechanical measurements, [17]. Our theorem shows that Bohr's original point of view reflects the physical side of the measurement process much better.

6. CONCLUSION

In conclusion, we will draw some conclusions and outline promising studies.

Wigner's elementary systems are the only mathematically and physically correct definitions of an elementary particle. But this definition depends entirely on the existence of a maximal group of motion. Maximal groups of motions exist only in the Minkowski world and de Sitter models. But in the general cosmological model, there are no maximal groups of motions. Thus, it is generally impossible to introduce the notion of an elementary particle. The concept of an elementary particle can be used only in the local sense. On the scale of a galaxy or a cluster of galaxies, the concept of an elementary particle is acceptable, because on such a scale Newtonian cosmology can be used in a post-Newtonian approach without applying general relativity. Einstein warned against introducing the concept of a particle into general relativity as early as 1916, [6]. Attempts to represent particles as field singularities cannot be considered satisfactory.

Here it is appropriate to note that in the general theory of relativity, black holes play the role of elementary particles, as in high-energy physics. Indeed, black holes, regardless of their origin, are characterized by only a few parameters, such as mass, angular momentum and electric charge, [11]. And in relativistic quantum mechanics, the basic characteristics of elementary particles, such as mass and spin, according to Wigner's theory, are invariants of kinematic groups of movements, whereas in general relativity, black hole parameters act as integration constants for solutions of Einstein equations. On the other hand, there is still no satisfactory explanation of the electric charge and lifetime of elementary particles.

Hawking showed in his paper [23] that black holes can have a finite lifespan due to quantum evaporation. We are confident that Hawking's work is the beginning of a new physics and will play the same role as Planck's work on blackbody radiation.

This is an example of the fact that the solution to the problems of fundamental physics will be found at the intersection of elementary particle physics and cosmology.

On the other hand, attempts to quantize the gravitational field have also failed, which implies that the synthesis of quantum mechanics with general relativity produces another physical object, differs from the relativistic quantum field of special relativity.

Determining the nature of this object depends on the solution of two problems. First, the reformulation of the principles of symmetry in the general theory of relativity exclusively in terms of invariant geometric objects. In this case, the symmetry properties of the cosmological vacuum play an essential role. Recall that from the constancy of the speed of light, it follows that the laws of physics in a vacuum must be invariant concerning a wider group than the inhomogeneous Lorentz group, namely, concerning the conformal group. H.Weyl wanted to use this fact to construct a unified geometric theory of electromagnetism and gravity but without success, [24, 25]. It's interesting that the Poincarè $ISO(3,1)$ group and de Sitter groups $SO(4,1)$, $SO(3,2)$ are globally subgroups of the conformal group $SO(4,2)$, [26].

Secondly, our theorem on the application of quantum mechanics to shows that it is necessary to find an adequate generalization of the Hamiltonian formalism to solve this problem. Preliminary results show that such a generalization is possible only if time is irreversible. This result leads to the following hypothesis:

The irreversibility of time is also a cosmological quantum effect.

In the context of our approach, a problem arises that supplements the well-known list of cosmological ones: Can cosmology provide a unique element of the equivalence class or the uniqueness of the nonequivalent classes of the representations of the algebra observed?

This is our program for further research.

This work is the basis of my report of the same name at the International Conference "Alive Universe - from Planets to Galaxies"[5)]. Therefore, I considered it necessary to include the questions put to me and my answers in the Appendix.

[5)] 12-14 October, 2022, Shamakhy, Y.Mammadaliyev settlement, ShAO, Azerbaijan

APPENDIX

A. Namig Dzhalilov, Dr.of Sci., Professor.

Question: *It is known that, in addition to mass and spin, elementary particles have different charges. What can you say about the charges?*

Answer: Charges come in different types. The first type of charge was introduced to classify elementary particles. For example, baryon, lepton and other charges. The second type of charge is a source of interaction. For example, an electric charge or color charge. The last type is mass, which is the source of gravitational interaction. Furthermore, according to Wigner, mass and spin are invariants of the kinematic group of motion, whereas for an electric charge there is no similar interpretation.

On the other hand, in general relativity, there are known solutions to Einstein's equations for black holes with mass, angular moment, and charge. Moreover, solutions with Dirac monopoly are also known. These quantities are obtained as integration constants for solutions to Einstein's equations. This is evidence that solutions to fundamental problems can be found after the synthesis of quantum mechanics with general relativity.

B. Janmammad Rustamov,Associate Prof. Dr.

Question: *Are the Void and cosmic vacuum the same?*

Answer: Not at all. The report is not about the cosmic vacuum, but about the cosmological one. The cosmological vacuum is a solution to Einstein's vacuum equations, (3). On the other hand, according to the standard cosmological model, the universe is homogeneous everywhere and isotropic in all directions, which is confirmed by the observed properties of relict radiation[6)]. And cosmic voids are local inhomogeneities on the scale of a cluster of galaxies, where general relativity is not needed, a post-Newtonian approximation is enough. And the hypothesis of dark matter in cosmic voids has not been confirmed, [32], [7)]. This proves once again that dark matter is not a local phenomenon, but a cosmological effect.

C. Nariman Ismailov, Dr.of Sci., Professor.

Question:*Currently, even not far from what it was 20 years ago, the field of cosmology is in a perplexed state. Various theories have been put forward,*

[6)] In 1992, the cosmological anisotropy of the cosmic microwave background was discovered.

[7)] I express my deep gratitude to Dr. J. Rustamov, who drew my attention to this site.

sometimes there are ideas that completely contradict each other. In this apparent confusion, what advantages does your cosmological model have, and how does it differ from others? How to get out of such a mess?

Answer: Let me be clear: I am not proposing a new cosmological model. There are standard cosmological models, and well-known models De Sitter, Friedman, Robertson-Walker, etc. Moreover, I propose to be based on the general theory of relativity and on the principles of quantum mechanics and avoid exotic theories such as superstring, multidimensional ones, various modifications of the Einstein's equation and etc. Using the de Sitter model as an example, I showed that, based on the Einstein equations and the concept of Wigner elementary systems, it is possible to explain the phenomena of dark matter and energy as quantum effects. Since these take place at a non-zero value of the cosmological constant, I have defined them as cosmological quantum effects.

The most difficult problem is to construct a representation of Jordanian algebra observed for interacting classical and quantum systems. Solving this problem will allow quantum mechanics to be applied to cosmology. I will prove that such a system cannot be Hamiltonian.

Two representations of the algebra of observables are known: classical and quantum mechanics. Two parameters characterize the equivalence classes of the representation: $\hbar$ and c. Experimental data show that representations with the following values are realized in Nature:

$$\hbar = 1.054\ 571\ 817... \times 10^{-34}\ \text{J}\cdot\text{s} = 6.582\ 119\ 569... \times 10^{-16}\ \text{eV}\cdot\text{s}$$
$$c = 299792458\ m/s.$$

Newtonian mechanics corresponds to the representation:

$$\hbar = 0; \qquad c = \infty.$$

I would like to point out the following. From a philosophical point of view, the observation algebra approach is a mathematical expression of the difference between subjective and objective. The choice of a particular representative of the representation of algebras observed within some non-equivalence class is subjective because it is our choice. But the element of the set of non-equivalent representations is determined by the laws of Nature through universal constants and therefore expresses the objective nature of scientific truths and is not dependent on our will. Whether cosmology limits our choices remains to be seen in future research.

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